
VI.
Galactic Accelerators and Acceleration
Mechanisms

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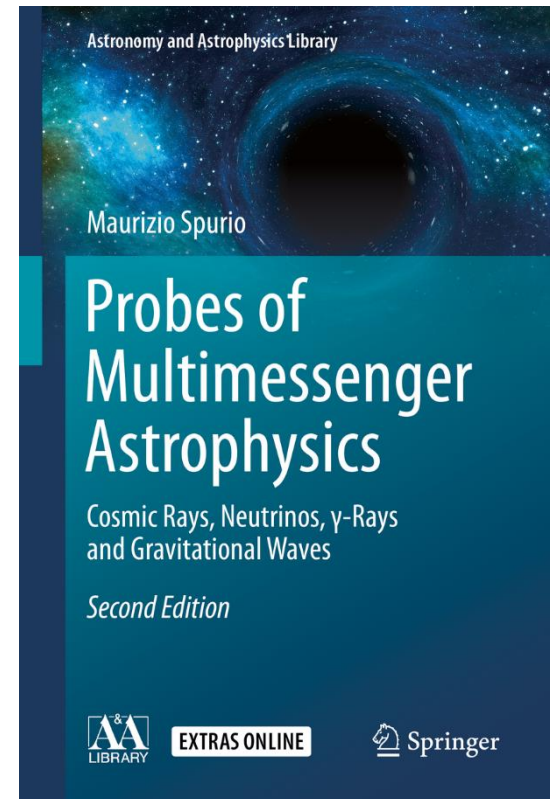
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The summary of experimental facts

The acceleration processes of CRs must explain the features observed in experimental data and discussed in the previous chapters, namely that:

- CRs have non-thermal energy distribution, and the energy spectrum has the form $\Phi(E) \propto E^{-\alpha}$ over a wide energy range;
- The measured exponent is $\alpha \sim 2.7$ for p and nuclei up to the knee region;
- The observed chemical abundances of CRs below the knee are similar to the abundances of elements found in our Solar System. This indicates that CRs are accelerated out of a sample of well-mixed interstellar matter.
- The observed exponent α becomes $\alpha \sim 3.1$ above the knee. The chemical composition seems to become heavier, although at such energies, no measurement of the mass number A of individual CRs is possible;
- After corrections for the effects due to the propagation in the Galaxy, the expected energy dependence near the sources is $Q(E) \propto E^{-2}$;
- Above 5×10^{18} eV, the energy spectrum flattens again to form the ankle; The Larmor radius of CRs above the ankle is larger than the galactic disk thickness;
- No preferred directions from the galactic plane are observed for the UHECRs. The origin of UHECRs is thus probably extragalactic

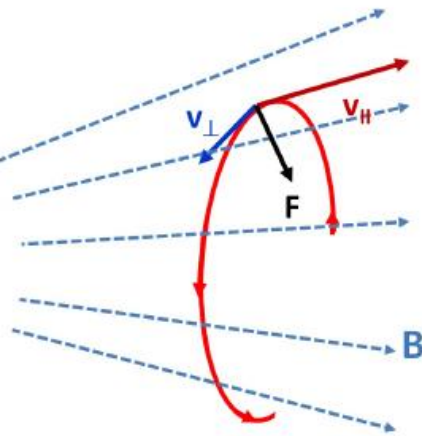
The possible acceleration mechanism(s)

- In astrophysical environments, **static electric fields cannot be maintained**, because matter is in the state of a plasma. Ionized gases have a high electrical conductivity, and any static electric field is rapidly short-circuited by the motion of free charges.
- CRs below the knee are thought to be accelerated by violent processes that produce shock waves and turbulences (SN explosions) .
- The bulk of CRs is believed to be accelerated in **recursive stochastic mechanisms** (described in this chapter) in which low-energy particles, after many interactions with a shock wave, reach high energies, up to 10^{15} – 10^{16} eV, with a spectral index $\alpha \sim 2$.
- This “standard model” of galactic CR acceleration has some limitations: in particular, it fails to describe the flux above the knee.
- Additional models have been put forward, such as the particle acceleration through EM mechanisms associated with time-varying magnetic fields (Faraday’s law).
- Some peculiar galactic objects can be involved in these processes, but at present, no firm experimental proof is evident for any point-like source of CRs.
- Because galactic magnetic field is not able to confine CRs with $E > 10^{18}$ eV, they are believed to be of extragalactic origin (Chap. 7)

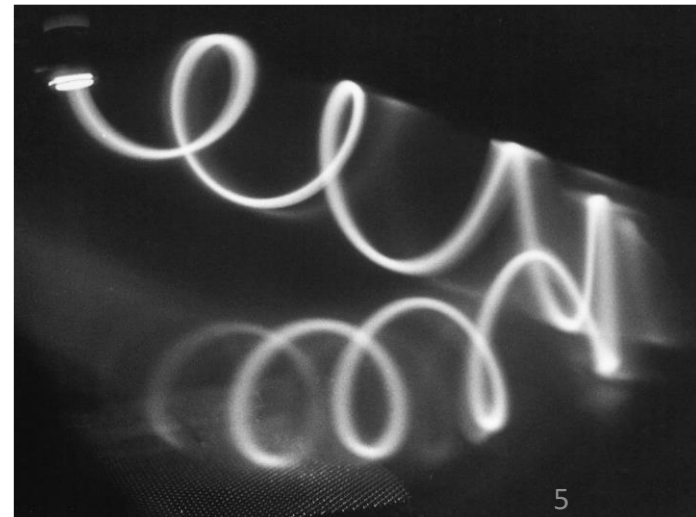
Magnetic Mirrors

- Acceleration of CRs can occur in regions where very strong inhomogeneous magnetic fields exist. The process can be seen as the scattering of the particle by magnetic field irregularities (**magnetic mirrors**, see figure).
- In literature, this situation is sometimes called a **collisionless shock**. A collisionless shock is defined as a shock wave in which the scattering occurs on a length scale much smaller than the mean free path λ (Eq. 3.1), necessary for a particle collision.
- In such a structure, particles interact with each other not through collisions, but by the emission and absorption of collective excitations of the plasma (plasma waves).
- In astrophysical situations, the magnetic field necessary to allow for the existence of such plasma waves, is provided by a “frozen” cloud of interstellar matter, with a density much higher than that of the surrounding material.

Motion of a charged particle in a nonuniform magnetic field



Picture of a mirror reflection of an electron beam in a magnetic field



The Fermi acceleration model

- One of the first models for CR acceleration was due to E. Fermi. He suggested a mechanism in which particles would be accelerated by collisions with moving clouds of gas.
- In each collision, a small fraction of energy is gained, $\Delta E = \beta E$, with $\beta \sim 10^{-2}$.
- The particles can reiterate the collision (having a finite escape probability from the acceleration region), increasing their energy and obtaining a power-law distribution

$$N(E) = KE^{-\alpha}$$

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On the Origin of the Cosmic Radiation

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(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

I. INTRODUCTION

IN recent discussions on the origin of the cosmic radiation E. Teller¹ has advocated the view that cosmic rays are of solar origin and are kept relatively near the sun by the action of magnetic fields. These views are amplified by Alfvén, Richtmyer, and Teller.² The argument against the conventional view that cosmic radiation may extend at least to all the galactic space is the very large amount of energy that should be present in form of cosmic radiation if it were to extend to such a huge space. Indeed, if this were the case, the mechanism of acceleration of the cosmic radiation should be extremely efficient.

I propose in the present note to discuss a hypothesis on the origin of cosmic rays which attempts to meet in part this objection, and according to which cosmic rays originate and are accelerated primarily in the interstellar space, although they are assumed to be prevented by magnetic fields from leaving the boundaries of the galaxy. The main process of acceleration is due to the interaction of cosmic particles with wandering magnetic fields which, according to Alfvén, occupy the interstellar spaces.

Such fields have a remarkably great stability because of their large dimensions (of the order of magnitude of light years), and of the relatively high electrical conductivity of the interstellar space. Indeed, the conductivity is so high that one might describe the magnetic lines of force as attached to the matter and partaking in its streaming motions. On the other hand, the magnetic field itself reacts on the hydrodynamics³ of the interstellar matter giving it properties which, according to Alfvén, can pictorially be described by saying that to each line of force one should attach a material density due to the mass of the matter to which the line of force is linked. Developing this point of view, Alfvén is able to calculate a simple formula for the velocity V of propagation of magneto-elastic waves:

$$V = H / (4\pi\rho)^{1/2}, \quad (1)$$

¹ Nuclear Physics Conference, Birmingham, 1948.

² Alfvén, Richtmyer, and Teller, *Phys. Rev.*, to be published.

³ H. Alfvén, *Arkiv Mat. f. Astr., o. Fys.* **29B**, 2 (1943).

where H is the intensity of the magnetic field and ρ is the density of the interstellar matter.

One finds according to the present theory that a particle that is projected into the interstellar medium with energy above a certain injection threshold gains energy by collisions against the moving irregularities of the interstellar magnetic field. The rate of gain is very slow but appears capable of building up the energy to the maximum values observed. Indeed one finds quite naturally an inverse power law for the energy spectrum of the protons. The experimentally observed exponent of this law appears to be well within the range of the possibilities.

The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.

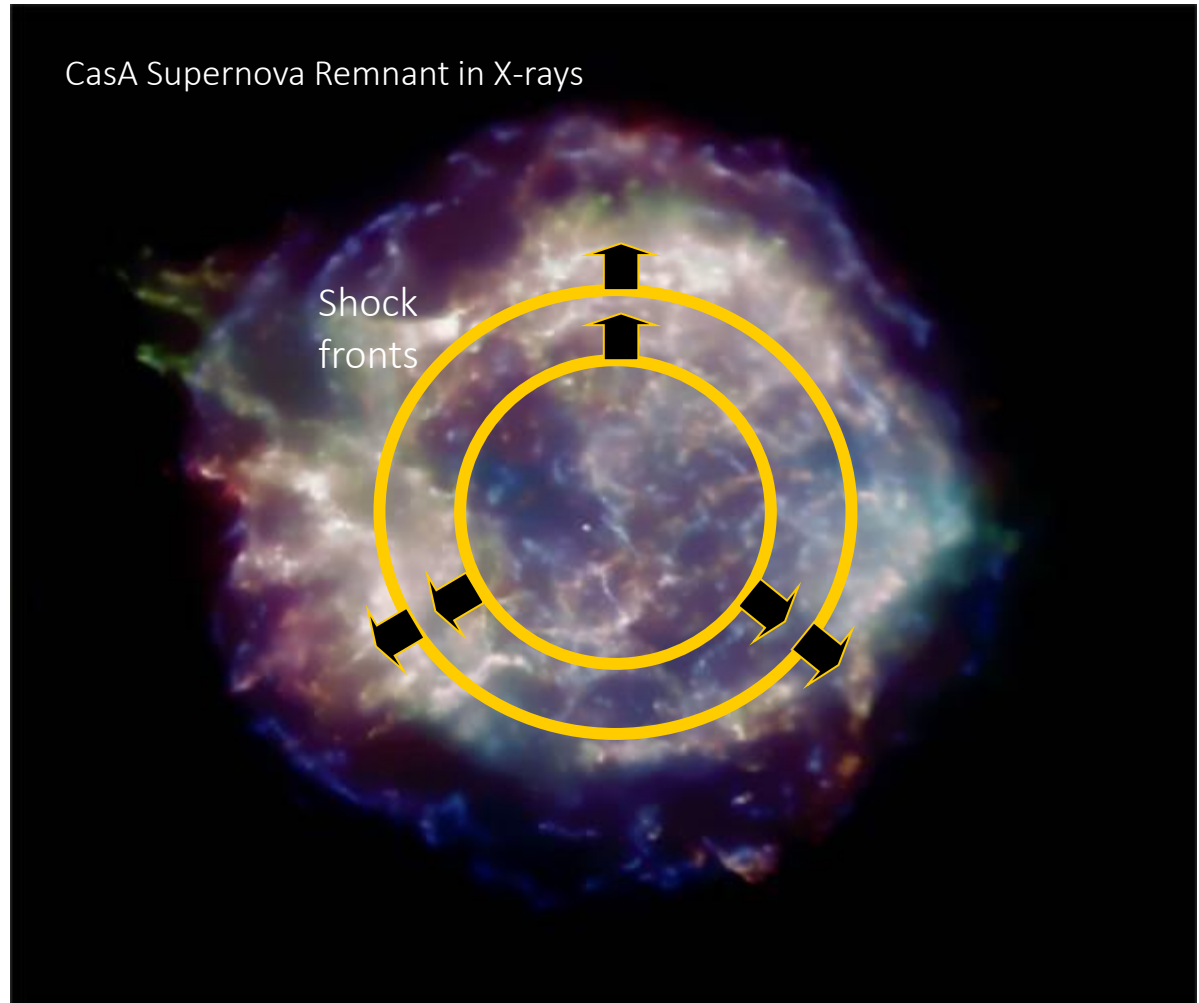
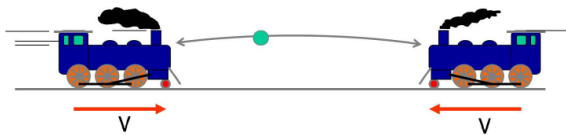
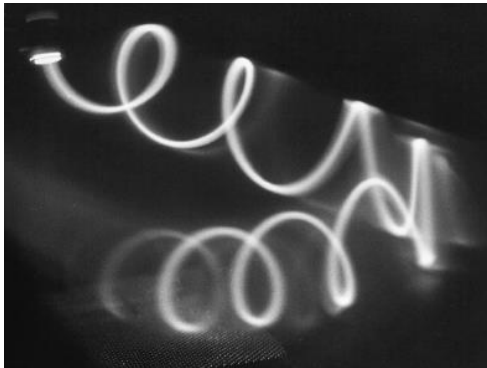
II. THE MOTIONS OF THE INTERSTELLAR MEDIUM

It is currently assumed that the interstellar space of the galaxy is occupied by matter at extremely low density, corresponding to about one atom of hydrogen per cc, or to a density of about 10^{-24} g/cc. The evidence indicates, however, that this matter is not uniformly spread, but that there are condensations where the density may be as much as ten or a hundred times as large and which extend to average dimensions of the order of 10 parsec. (1 parsec. = 3.1×10^{14} cm = 3.3 light years.) From the measurements of Adams⁴ on the Doppler effect of the interstellar absorption lines one knows the radial velocity with respect to the sun of a sample of such clouds located at not too great distance from us. The root mean square of the radial velocity, corrected for the proper motion of the sun with respect to the neighboring stars, is about 15 km/sec. We may assume that the root-mean-square velocity

⁴ W. S. Adams, *A. J.* **97**, 105 (1943).

The Fermi acceleration model: ingredients

- Shock wave +
- Magnetic fields +
- Iterative processes



Kinematics of the scattering process

- S = reference frame of the observer, with the cloud speed U directed along the x -axis;
- S' = the frame in which the cloud is at rest.

Only the p_x component is relevant, as the y ; z components are conserved in the interaction. The four-momentum describing the particle is $(E; p_x)$ in the frame S , and $(E'; p'_x)$ in S' . Thus

$$E' = \Gamma(E + Up_x) , \quad (6.12a)$$

$$p'_x = \Gamma(p_x + \frac{U}{c^2}E) . \quad (6.12b)$$

The collision is elastic in the S' frame.

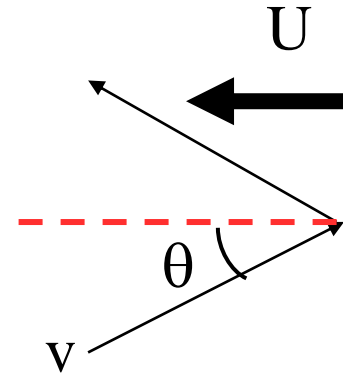
$$E' \xrightarrow{\text{collision}} E'; \quad p'_x \xrightarrow{\text{collision}} -p'_x . \quad (6.13)$$

$$E = \Gamma(E' - Up'_x) \xrightarrow{\text{collision}} \Gamma(E' - U(-p'_x)) \equiv E^* . \quad (6.14)$$

$$E^* = \Gamma \left[\Gamma(E + Up_x) + U\Gamma(p_x + \frac{U}{c^2}E) \right] . \quad (6.15)$$

Recalling that $p_x = mv\Gamma \cos \theta$ and $E = mc^2\Gamma$

$$\frac{p_x}{E} = \frac{mv\Gamma \cos \theta}{mc^2\Gamma} = \frac{v}{c^2} \cos \theta , \quad (6.16)$$



Kinematics of the scattering process

Thus, the energy E^* (6.15) after the collision becomes

$$E^* = \Gamma^2 \left[E + 2U p_x + \frac{U^2}{c^2} E \right] = \Gamma^2 E \left[1 + 2U \frac{p_x}{E} + \frac{U^2}{c^2} \right] = \Gamma^2 E \left[1 + 2U \frac{v \cos \theta}{c^2} + \frac{U^2}{c^2} \right] \quad (6.17)$$

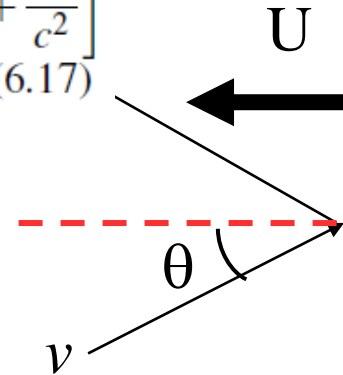
And, at the second order of U/c :

$$E^* \simeq \left[1 + \frac{U^2}{c^2} \right] E \left[1 + 2U \frac{v \cos \theta}{c^2} + \frac{U^2}{c^2} \right] \simeq E \left[1 + 2 \frac{U v \cos \theta}{c^2} + 2 \frac{U^2}{c^2} \right]$$

The energy gained by the particle in the **observer reference frame** is:

$$\Delta E = E^* - E = \left[2 \frac{U v \cos \theta}{c^2} + 2 \left(\frac{U}{c} \right)^2 E \right]. \quad (6.19)$$

- The first term in (6.19) is null when averaged over all directions. If this is done, we have a gain $\propto (U/c)^2$ (**second-order Fermi model**). The acceleration process is therefore rather inefficient.
- Energy is gained in head-on collisions ($\cos \theta > 0$) and lost in catching collisions when $\cos \theta < 0$.
- The **first-order Fermi model** considers head-on collisions only.



Head-on in first-order Fermi model



- At 1st order for a relativistic particle, (6.19) becomes

$$\Delta E = E^* - E = \left(2 \frac{U \cos \theta}{c}\right) \cdot E;$$

- Averaging over all head-on direction ($\cos \theta > 0$)

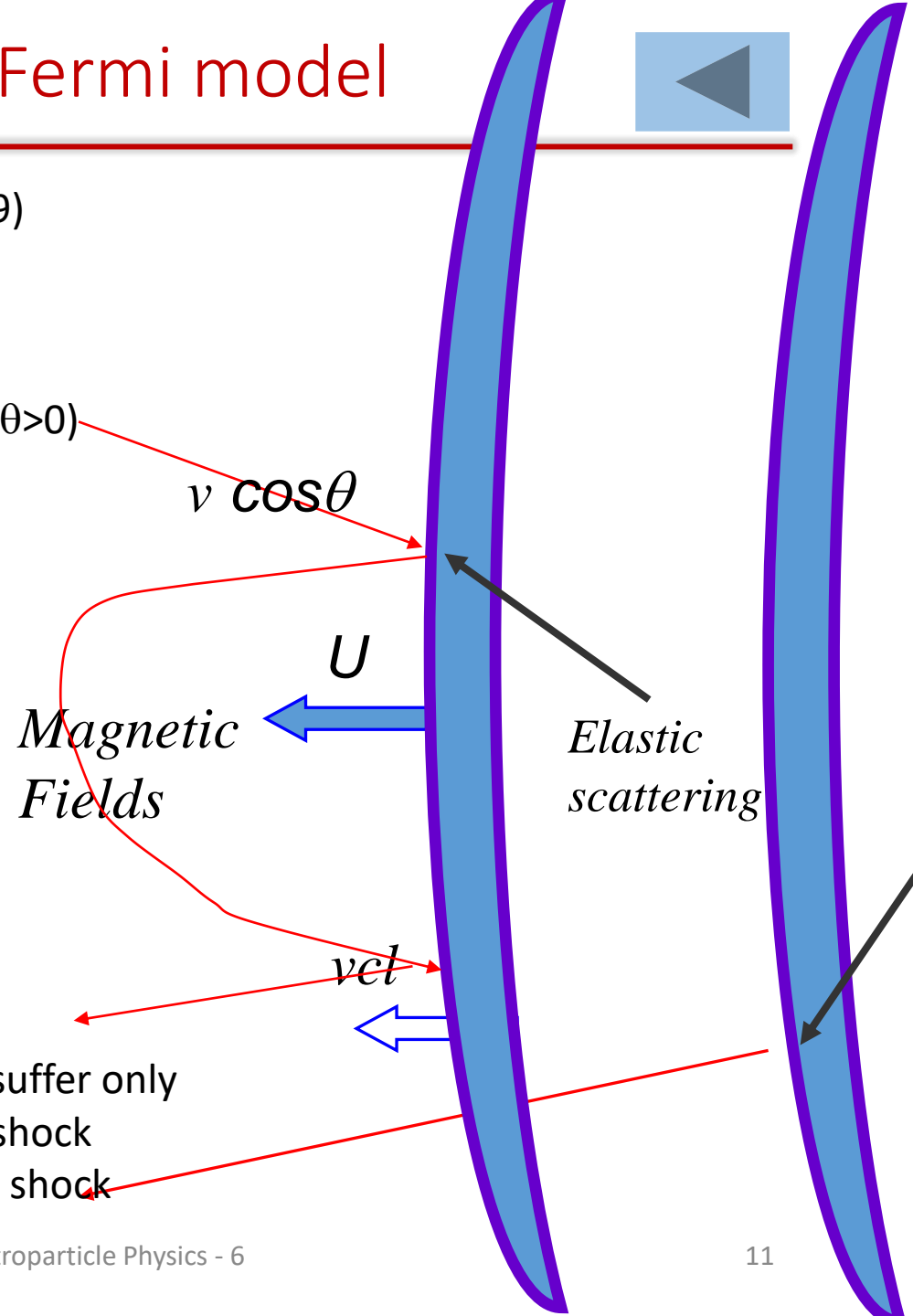
$$\langle x \rangle = \frac{\int x \cdot f(x) \cdot d\Omega}{\int f(x) \cdot d\Omega}.$$

$$\langle \cos \theta \rangle = \frac{\int_0^1 \cos \theta \cdot \cos \theta \cdot d \cos \theta}{\int_0^1 \cos \theta \cdot d \cos \theta} = \frac{2}{3}$$

(6.23a) $\langle \Delta E \rangle = \left(\frac{4U}{3c}\right) \cdot \langle E \rangle \equiv \eta \cdot \langle E \rangle,$

(6.23b) $\langle E^* \rangle = \left(1 + \frac{4U}{3c}\right) \cdot \langle E \rangle \equiv \mathcal{B} \cdot \langle E \rangle.$

A situation in which accelerated particles suffer only head-on collisions is assumed in diffusive shock acceleration model, which uses the strong shock waves produced by SNRs.



The Power-Law Spectrum from Fermi Model

- From (6.23), the particle gains energy in each head-on collision, on average $E^* = \mathcal{B}E_0$ and there is a finite probability, P , that the particle remains in the acceleration region.
- After k collision, we have:

- Average energy of particles after k collision: $E = \mathcal{B}^k E_0$

- Number of particles with energy E : $N(> E) = P^k N_0$

- It is easy to show that:
$$\frac{\ln\left(\frac{N(>E)}{N_0}\right)}{\ln\left(\frac{E}{E_0}\right)} = \frac{\ln P}{\ln \mathcal{B}} \equiv \alpha$$

- and thus:
$$\frac{N(>E)}{N_0} = \left(\frac{E}{E_0}\right)^\alpha$$

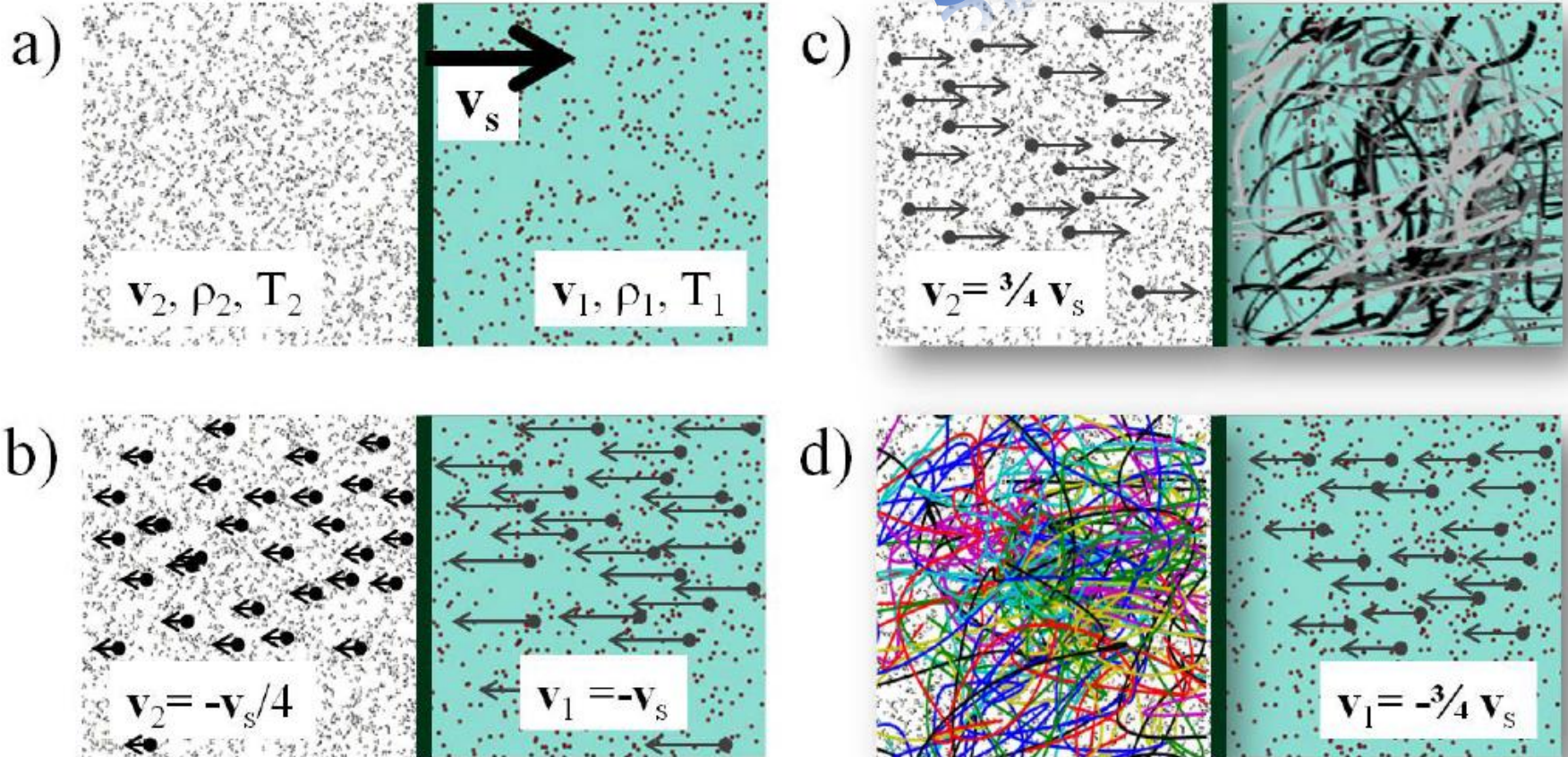
- This represents the (integral) number of particles with energy larger than E , and :

$$\frac{dN}{dE} = N_0 \left(\frac{E}{E_0}\right)^{\alpha-1}$$

- This is the power-law energy spectrum derived in the Fermi model.
- It is possible to derive also that $\alpha \cong -1$ (refer for details to Sect. 6.5.1)
- Thus, near sources, the spectral index of the differential flux is $dN/dE \sim E^{-2}$

The spectral index $(\alpha-1)=-2$ (§6.5.1, 6.5.2)

Skip this



SNRs and Standard Model of CRs Acceleration

- There is a consensus in the community that Galactic CRs are somehow related to one or more types of supernova explosion
- Acceleration process is mainly due to diffusive transport in the neighborhood of strong shocks formed as a consequence of these explosions.
- The environment provided by the shock wave produced by Type II, or core collapse supernovae (SNe), is the main candidate.
- **Type II SNe occur at the end of the fusion process in very massive stars, $M > 8M_{\odot}$.**
- The experimental facts in support of this are:

- ▶ • **The energy balance** between CR escaped energy and SN energy release (Chap.2)
- ▶ • **The chemical abundances** of CRs and SN environments (see Chap. 3 and 5)
- ▶ • **The spectral index of the power-law** energy spectrum (Chap. 5).
 - **The required efficiency** of the acceleration process (see below)
 - **The maximum energy attainable** by the model (see below)

Relevant Quantities in SNRs

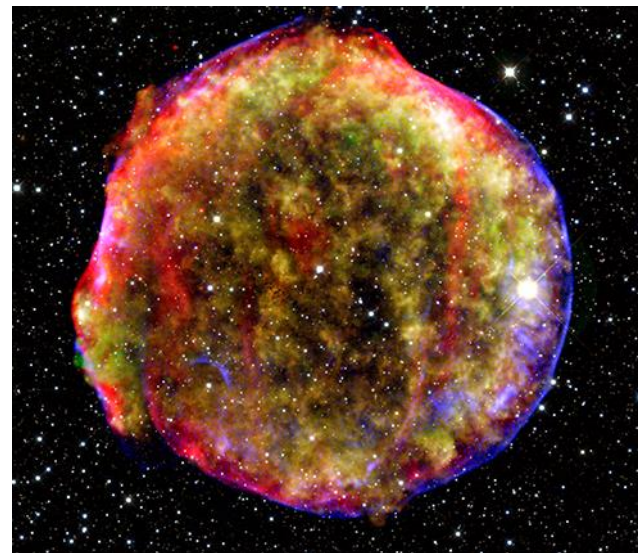
- The observed emitted kinetic energy, K , by a $10 M_{\text{sun}}$ supernova is roughly 1% of the total binding energy, i.e., $K = 2 \times 10^{51}$ erg.
- The gravitational binding energy is 2×10^{53} erg.
- The velocity U of the ejected mass (the shock wave)

$$(6.33) \quad U \simeq \sqrt{\frac{2K}{M}} = \sqrt{\frac{4 \times 10^{51}}{2 \times 10^{34}}} \simeq 5 \times 10^8 \text{ cm/s} \quad \rightarrow \quad \frac{U}{c} \simeq 2 \times 10^{-2}$$

- Some regions can have higher velocities, $U/c \sim 0.1$
- The needed efficiency η of the acceleration process defined in (6.23a) and required to explain the CRs' acceleration by supernovae explosions corresponds to:

$$\frac{4}{3} \frac{U}{c} \equiv \eta \simeq 10^{-2} - 10^{-1}$$

- The shock front expands across the ISM. During the expansion, the shock collects interstellar matter. When the mass of the swallowed material becomes comparable to the mass of the ejected shells of the SN, the velocity of the shock decreases remarkably.
- As the radius R_{SN} of the shock increases, the matter density ρ_{SN} decreases.
- The shock becomes inefficient for acceleration when the $\rho_{\text{SN}} = \rho_{\text{ISM}}$.



Relevant Quantities in SNRs

- The radius within which the shock wave is able to accelerate particles can be derived using the condition

$$\rho_{\text{SN}} = \frac{10M_{\odot}}{(4/3)\pi R_{\text{SN}}^3} = \rho_{\text{ISM}}$$

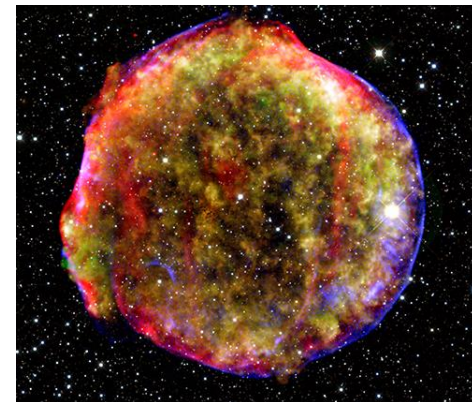
$$(6.36a) \quad R_{\text{SN}} = \left(\frac{3 \times 10M_{\odot}}{4\pi\rho_{\text{ISM}}} \right)^{1/3} = \left(\frac{6 \times 10^{34}}{4\pi \cdot 1.6 \times 10^{-24}} \right)^{1/3} = 1.4 \times 10^{19} \text{ cm} = 5 \text{ pc} .$$

$$(6.36b) \quad T_{\text{SN}} = \frac{R_{\text{SN}}}{U} = \frac{1.4 \times 10^{19} \text{ cm}}{5 \times 10^8 \text{ cm/s}} \simeq 3 \times 10^{10} \text{ s} \simeq \mathcal{O}(1000) \text{ y} .$$


Exercise: Figure shows the **SN 1572** (or Tycho's Supernova) as seen in different wavelengths by modern astronomers. It occurred in the Cassiopeia constellation, about **$D = 3 \text{ kpc}$** from Earth. SN 1572 is one of about eight SN visible to the naked eye in historical records. Its **angular radius is 4 arcmin**.

- Derive the linear radius of the object visible from the figure
- Derive the speed of the ejected material

For the answer, use the information in **bold**.



Maximum Energy in the Supernova Model

- With simple arguments, it is possible to derive the maximum energy that CR can reach in the acceleration process due to the diffusive shock mechanism.
- Eq. (6.23a) gives the energy gain for each iterative acceleration process. 
- The energy increase at a rate given by the ratio between (6.23a) and the characteristic period T_{cycle} :
$$\frac{dE}{dt} \simeq \frac{\eta E}{T_{\text{cycle}}}$$
- The typical extension of the confinement region is given by the Larmor radius in the magnetic field B:
$$\lambda_{\text{cycle}} = r_L = \frac{E}{ZeB}$$
- and thus the typical period T_{cycle} of the process:
$$T_{\text{cycle}} = \frac{\lambda_{\text{cycle}}}{U} = \frac{E}{ZeBU}$$
- Consequently, the rate of energy gain is independent of the initial energy:
$$\frac{dE}{dt} \simeq \eta E \frac{ZeBU}{E}$$
- The maximum energy that a charged particle could achieve is then simply the rate of energy gain, times the duration T_{SN} of the shock:

$$E^{\text{max}} \simeq \frac{dE}{dt} \times T_{\text{SN}} = \eta ZeBR_{\text{SN}} \simeq \frac{ZeBR_{\text{SN}}U}{c}$$

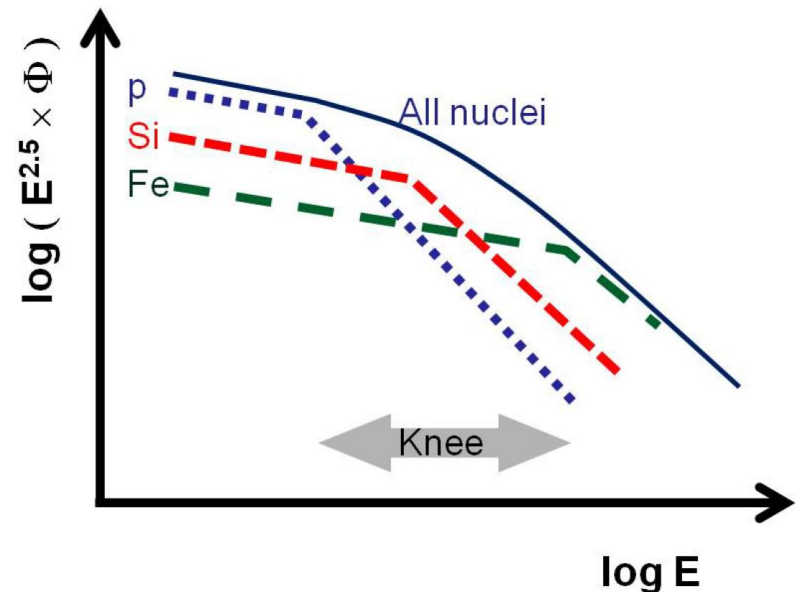
Maximum Energy in the Supernova Model

- Inserting the numerical values for the velocity U of the shock (6.33), the proton electric charge, the maximum radius of the expansion R_{SN} (6.36a), and the typical value of the galactic magnetic field B , we obtain

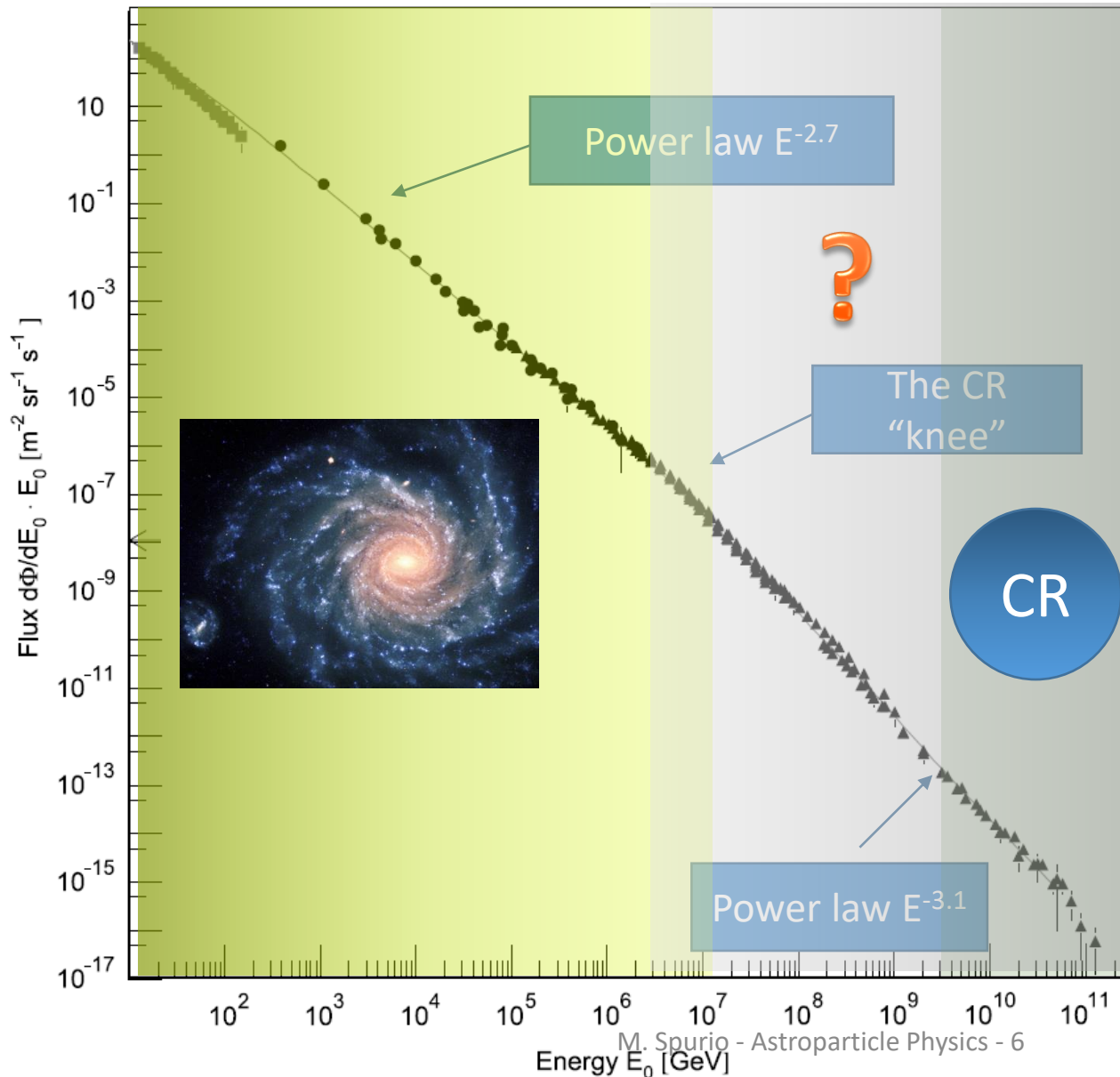
$$E^{\text{max}} = eB(U/c)R_{\text{SN}}Z \quad (6.43a)$$

$$\begin{aligned} &= (4.8 \times 10^{-10}) \cdot (4 \times 10^{-6}) \cdot (2 \times 10^{-2}) \cdot (1.4 \times 10^{19}) \cdot Z \\ &\simeq 500 \cdot Z \text{ erg} \simeq 300 \cdot Z \text{ TeV} . \end{aligned} \quad (6.43b)$$

- This mechanism explains the spectrum of CR protons up to a few hundred TeV;
- This corresponds to the *knee* energy
- An important consequence of (6.43) is that E_{max} depends on the particle charge Z .
- A heavy nucleus Z could achieve much higher total energy with respect protons.
- The knee is explained as a structure resulting from the different maximum energy reached by nuclei with different charge Z (see fig)



The sources of CRs (Chap. 2)



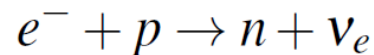
Galactic (SuperNova Remnants- SNRs)

ExtraGalactic (AGN, GRBs)

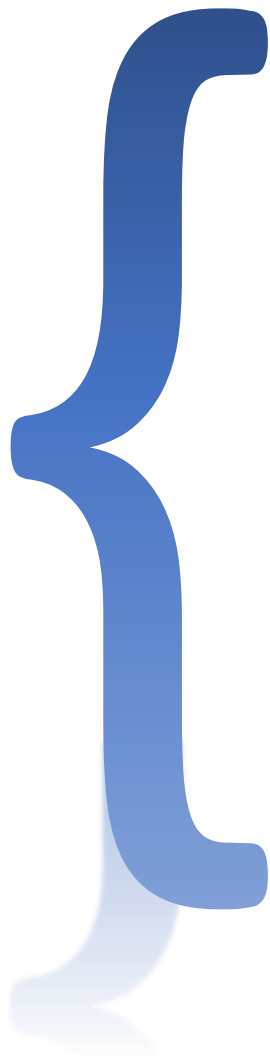


Higher energy accelerators

- In order to overcome the rigidity-dependent limit (6.43), the possibility has been proposed that, under particular conditions, cosmic rays suffer additional acceleration by **variable magnetic fields** in the acceleration region.
- Candidate that can accelerate CRs up to a maximum energy of 10^{19} eV are galactic compact objects, as
 - **neutron stars** (the remnant of the final stages of a massive star) or
 - a **binary system** (a neutron star or a black hole and a companion star)
- A **neutron star (NS)** is a stellar remnant that can result from the gravitational collapse of a massive star ($M > 8_{\text{sun}}$). As the core of a massive star is compressed during a supernova event, increases in the electron energy allow the reaction:



- Neutrinos escape, the matter cools down, the density increases and nuclei in the center of the star become neutron-enriched.



NS and BH

Neutron Stars (in one page)

Quantum Physics

$$\Delta p \cdot \Delta x \approx \hbar$$

- number density $n [\text{cm}^{-3}] = \frac{1}{d^3}$

- d = distance among baryons.

- Typical momentum

$$\Delta p \cdot \Delta x \approx \hbar \rightarrow p \cdot d \approx \hbar$$

$$\text{or } p \approx \frac{\hbar}{d}$$

- Degenerated energy of the baryons:

$$E_d \approx pc \approx \frac{\hbar c}{d} \approx \hbar c \cdot n^{1/3}$$

Gravity

Mass M , radius R

$$U = \frac{3}{5} \frac{GM^2}{R}$$

- number of baryons of mass $m_p \Rightarrow N = \frac{M}{m_p}$

- number density $\Rightarrow n [\text{cm}^{-3}] = \frac{N}{V} = \frac{N}{R^3} = \frac{M}{m_p R^3}$

- Total degenerated energy of N baryons

$$E_{\text{deg}} = N \cdot E_d = \left(\frac{M}{m_p}\right) \cdot (\hbar c n^{1/3})$$

$$= \left(\frac{M}{m_p}\right) \cdot (\hbar c) \cdot \left(\frac{M}{m_p R^3}\right)^{1/3}$$

$$= \hbar c \frac{M^{4/3}}{m_p^{4/3} R}$$

- The total degenerated energy is due to the gravitational energy: $|U| = E_{\text{deg}}$

$$\frac{GM^2}{R} \approx \hbar c \frac{M^{4/3}}{m_p^{4/3} R} \quad (R \text{ disappear!}) \Rightarrow M^{2/3} = \frac{\hbar c}{G m_p^{4/3}} = \frac{\hbar c}{G m_p^{6/3}} \cdot m_p^{2/3} \rightarrow M = \left(\frac{\hbar c}{G m_p^2}\right)^{3/2} \cdot m_p$$

- The mass of the system is given by a dimensional quantity $\alpha_G \equiv \frac{G m_p^2}{\hbar c} \approx 6 \times 10^{-39}$

$$M \approx \frac{m_p}{\alpha_G^{3/2}} \approx 3.4 \times 10^{30} \text{ kg} \approx 1.7 M_\odot$$

N.B. The correct value of the Chandrasekhar mass is $1.4 M_\odot$

Neutron Stars (in one page)

The mass of the system is given in terms of a dimensionless quantity

$$\alpha_G \equiv \frac{Gm^2}{\hbar c} \approx 6 \times 10^{-39}$$

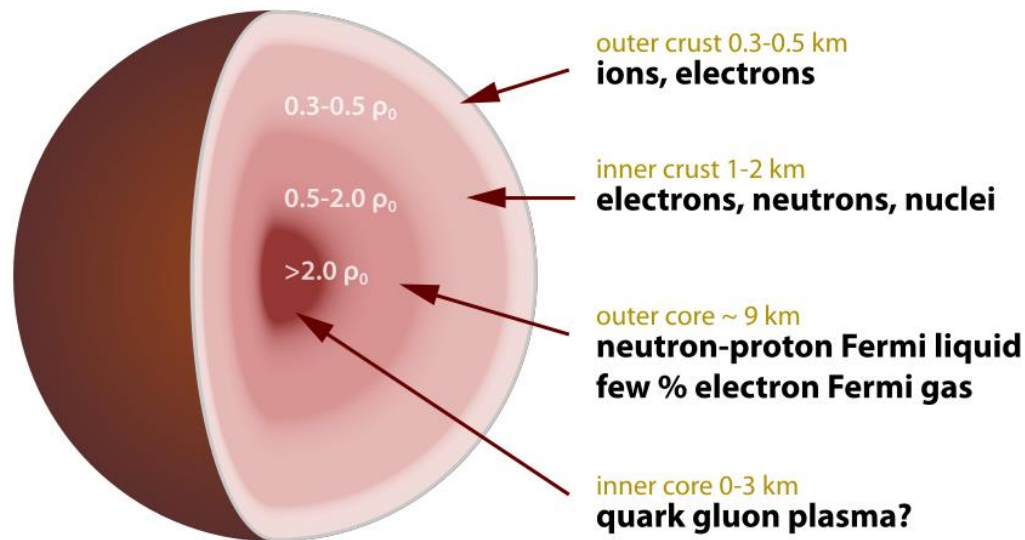
and the proton mass, m_p :

$$M \approx \frac{m_p}{\alpha^{3/2}} \approx 3.4 \times 10^{30} \text{ kg} \approx 1.7 M_\odot$$

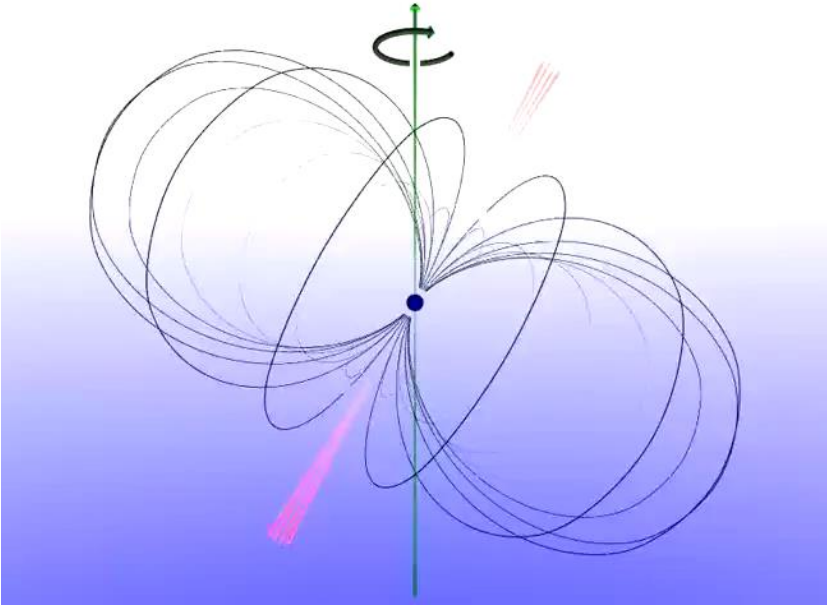
- NB: The value of this value (the “Chandrasekhar mass”) is $1.4 M_{\text{sun}}$

Higher energy accelerators

- A NS is an object with a defined mass, $M_{\text{NS}} \sim 1.4M_{\text{sun}}$, the Chandrasekhar mass
- Its density corresponds to the density of a nucleus, 10^{14} g/cm^3 ,
- The radius of a neutron star is thus a few km (**exercise**)
- If the remnant star has a mass greater than the Chandrasekhar limit, it continues collapsing to form a **black hole**, Sect. 6.8.
- The details of their structure is unknown (the Equation of state, EoS)
- The maximum observed mass of neutron stars is $\sim 2.0M_{\text{sun}}$



Pulsars

- A pulsar is a rotating neutron star that emits a beam of electromagnetic radiation, typically along its magnetic axis.
 - They were discovered in the radio band by Hewish and Bell in 1967, and soon identified with isolated, rotating, magnetized NSs.
 - The key observations were the very stable, short periods of the pulses and the observation of polarized radio emission.
 - The radiation can only be observed when the axis is pointing towards the Earth.
- 
- The diagram illustrates a pulsar as a rotating neutron star. A central blue dot represents the star, with a vertical green line indicating its axis of rotation. A curved arrow at the top of the axis shows the direction of rotation. Two large, overlapping loops of black lines represent the magnetic field lines, which are tilted relative to the rotation axis. A bright pink beam of radiation is shown originating from the magnetic poles and sweeping across the sky as the star rotates. A smaller red beam is also visible on the opposite side.
- More than 2,600 pulsars are known and present in the catalogue.
<http://www.atnf.csiro.au/people/pulsar/psrcat/>
 - The rotation period, and thus the interval between observed pulses, is very regular, and the periods of their pulses range from 1.4 ms to 8.7 s.
 - This rotation slows down over time as electromagnetic radiation is emitted

Pulsar properties

- The ms period for young pulsars can be estimated using basic physics arguments.
- A star (M_{sun}) has radius $R \sim 7 \times 10^5$ km and it rotates at 1 revolution/30 days, so that the angular velocity is $\omega \cong 2.5 \cdot 10^{-6}$ rad/s.
- After collapse, $R_{\text{NS}} \cong 10$ km and from angular momentum conservation:

$$MR^2\omega = MR_{\text{NS}}^2\omega_{\text{NS}},$$

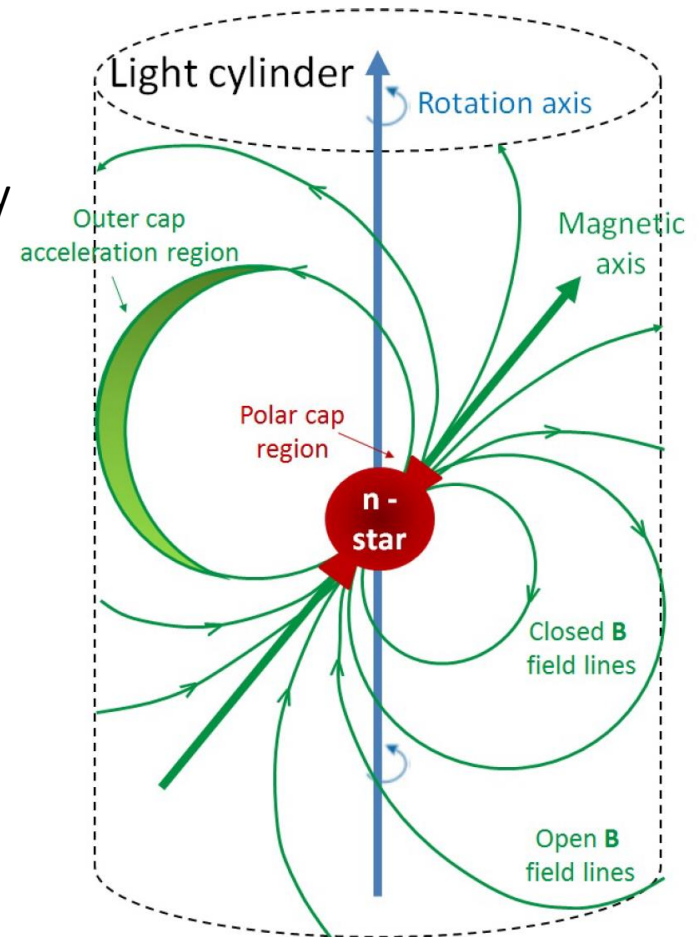
$$\omega_{\text{NS}} = \left(\frac{R}{R_{\text{NS}}}\right)^2 \times \omega = 12,500 \text{ rad/s}, \quad (6.79)$$

- The collapse amplifies the stellar magnetic field as well:

$$\oint \mathbf{B}_{\text{star}} \cdot d\mathbf{A}_{\text{star}} = \oint \mathbf{B}_{\text{NS}} \cdot d\mathbf{A}_{\text{NS}}$$

$$4\pi B_{\text{star}} R^2 = 4\pi B_{\text{NS}} R_{\text{NS}}^2 \rightarrow B_{\text{NS}} = B_{\text{star}} \frac{R^2}{R_{\text{NS}}^2}.$$

- For typical values $B_{\text{star}} = 1,000$ Gauss, the magnetic fields on the NS surface become on the order of $B_{\text{NS}} = 10^{11} - 10^{12}$ Gauss.



Stellar Mass Black Holes

- A **black hole (BH)** is a massive object exhibiting such strong gravitational effects that nothing (particles and electromagnetic radiation) can escape from inside its boundary, called the **event horizon**.
- In most cases, the event horizon corresponds to the **Schwarzschild radius** (exact for non-rotating massive objects that fit inside this radius).
- The Schwarzschild radius can be derived classically (escape velocity)

$$\frac{1}{2}mv^2 = G \frac{mM}{r}$$

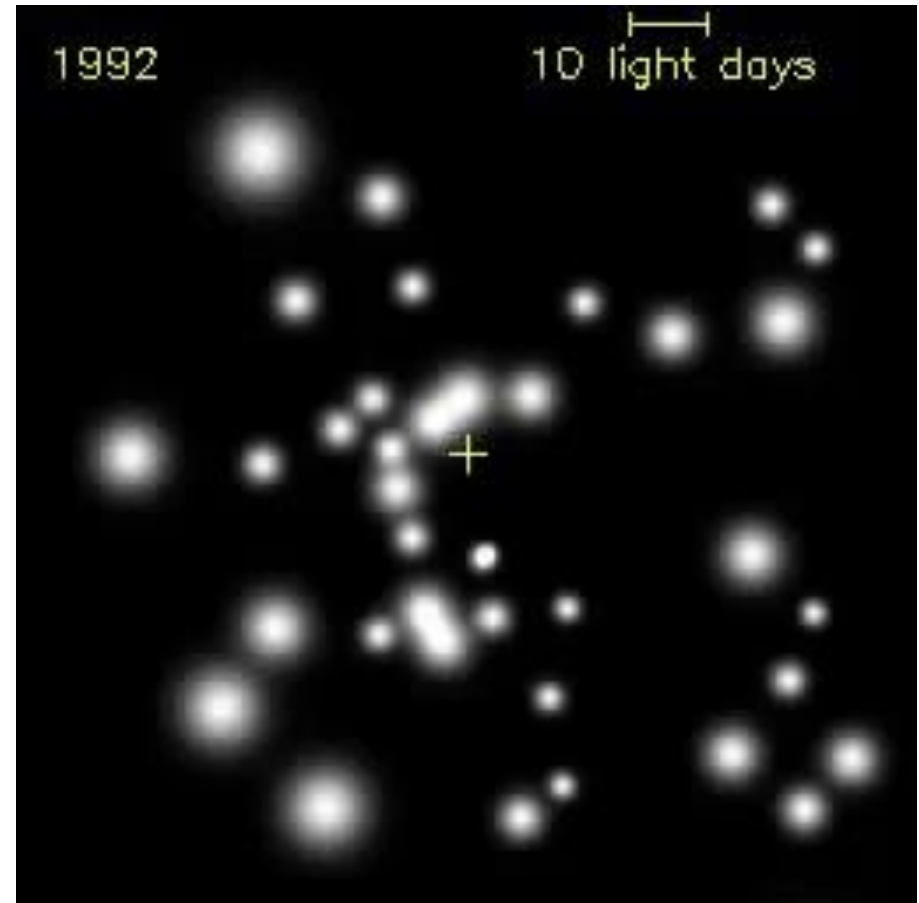
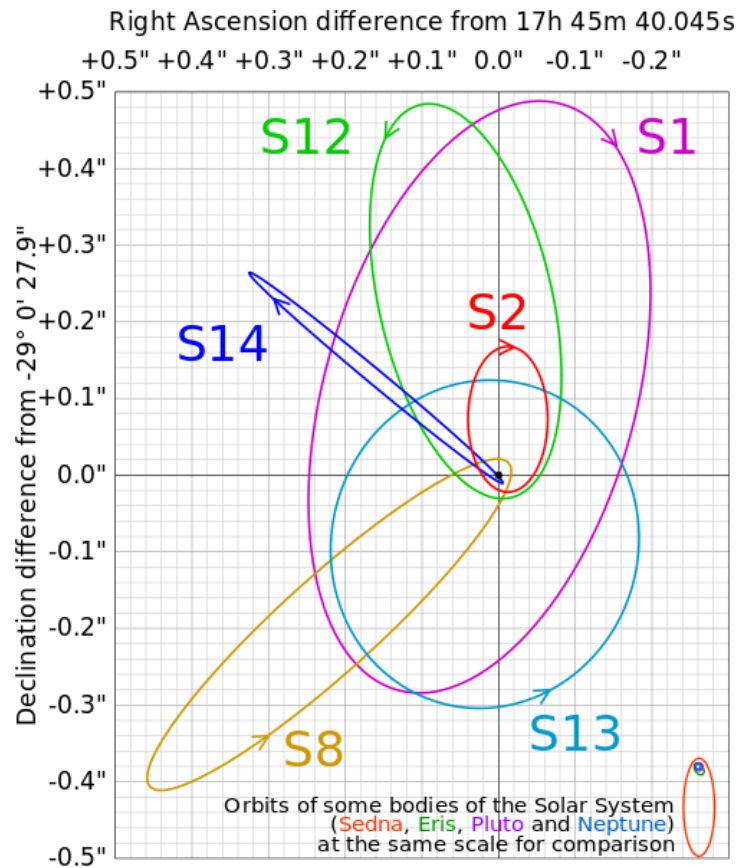
The limit speed ($v=c$) from “m” is reached for a radius:

$$\mathcal{R} = \frac{2GM}{c^2} = 2.95 \left(\frac{M}{M_{\odot}} \right) km$$

- This quantity scales linearly with the object mass. If the body is sufficiently dense and confined within \mathcal{R} , the Schwarzschild radius represents its event horizon and its inner region behaves like a black hole. Particles and light can escape the black hole only if they remain outside the event horizon.
- By absorbing other stars and merging with other black holes, supermassive BH of millions of solar masses may form.
- It is likely that supermassive black holes exist in the centers of most galaxies.

Did you ever see a SMBH? (Cap. 2)

- Exercise: evaluate the BH mass

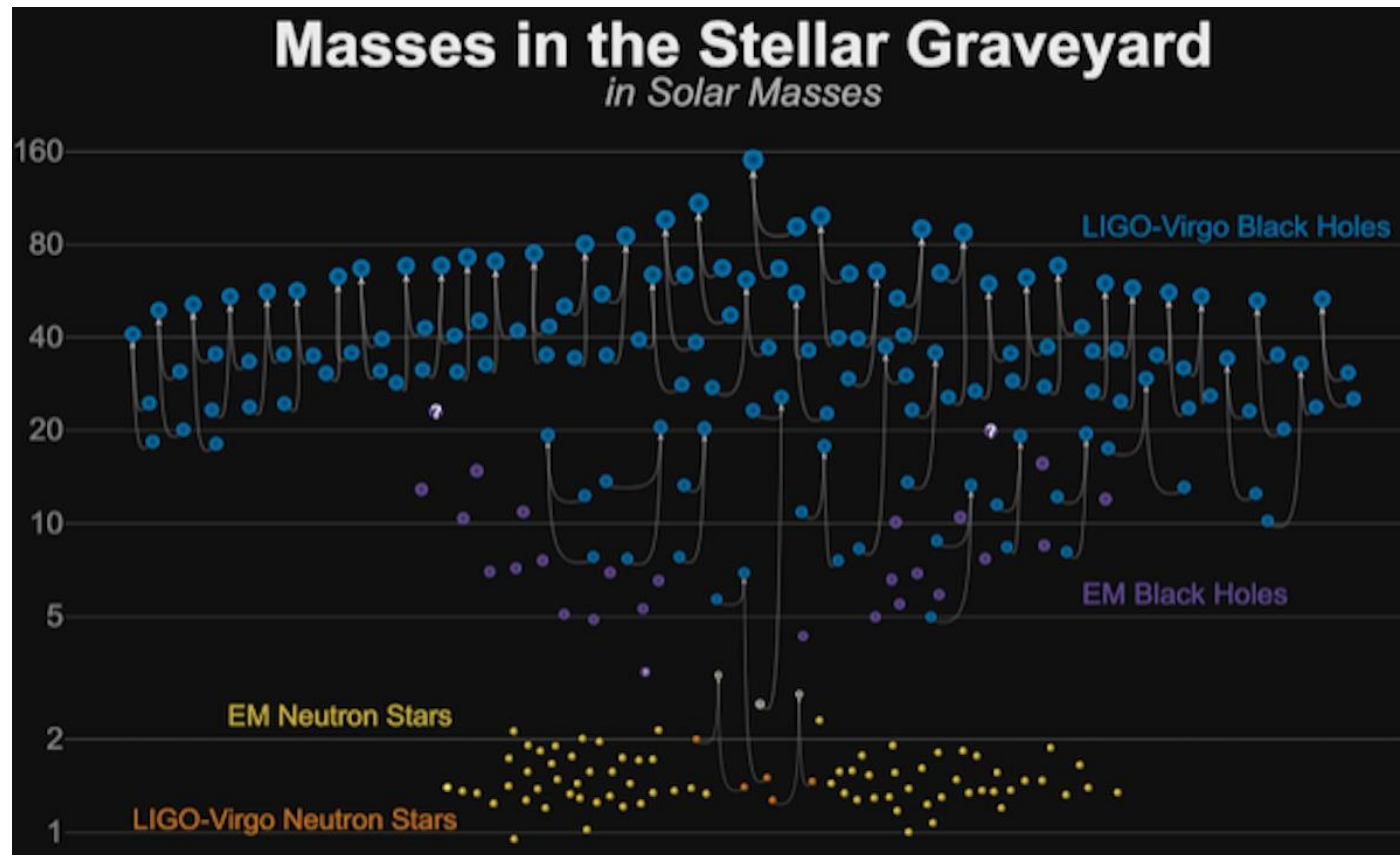


Stellar BH: observations

- Although we derived the Schwarzschild radius from Newtonian considerations, the same conclusion emerges from general relativity.
- More generally, BH are particular solutions to the Einstein field equations.
- It has been demonstrated (**no-hair theorem**) that a stable BH is completely described at any time by the following quantities:
 - its mass-energy, M ;
 - its angular momentum, or spin, \mathbf{S} (three components);
 - its total electric charge, Q .
- The number of stellar black holes in our Galaxy is unknown, but there are several stellar-mass black hole candidates.
- BHs do not emit radiation but, under particular conditions, we can discover their presence.
- Stellar black holes in close binary systems are observable when matter is transferred from a companion star to the black hole. The energy release by matter falling toward the compact object is so large that it **radiates in X-rays**. The BH (outside \mathcal{R}) is observable in X-rays, whereas the companion star can with optical telescopes.
- Until 2016, there were ~ 20 stellar BHs indirectly detected in such a way. The largest of them was $\sim 15M_{\text{sun}}$; the more likely mass was $5-10 M_{\text{sun}}$.

Stellar BH: observations after September 2015


- On September 14th, 2015, the LIGO gravitational wave observatory made the first-ever successful observation of gravitational waves.
- Starting from this observation (GW150914), the astrophysical models of stellar evolution probably should be revised (we return on that in Chapter 13)



<https://www.youtube.com/watch?v=nawilfudYuA>



Possible Galactic CR Sources above the Knee

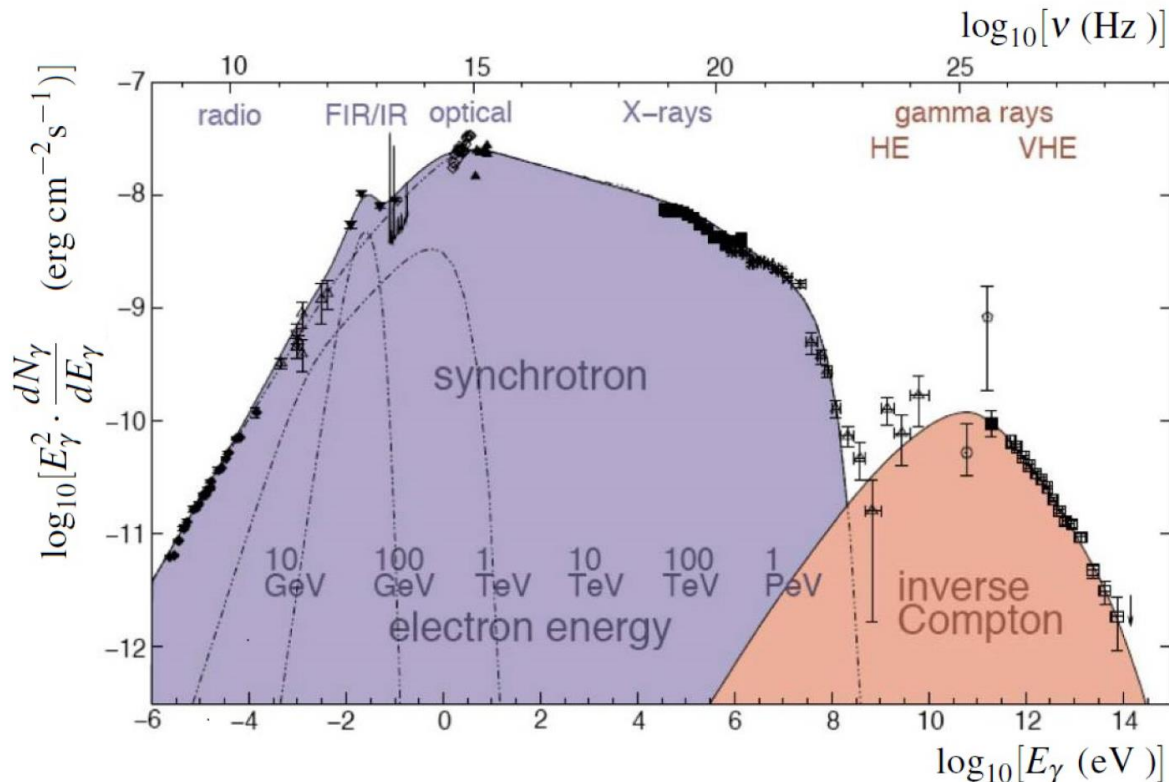
- **Dimensional arguments** to obtain the maximum attainable energy of a charged particle near an astrophysical object with a strong, rotating magnetic field.
- **Remember:** for the whole CR spectrum, we need order of 10^{40} - 10^{41} erg/s 
- The power needed to accelerate CRs above the knee is three orders of magnitude smaller than that required for the whole CR spectrum (*not shown. We need the diffusion equation, skipped in Chap. 5. See also Sect. 6.9*)

$$\begin{aligned} P(> E_0) &\simeq 2 \times 10^{39} \text{ erg/s for } E > 100 \text{ TeV ,} \\ &\simeq 2 \times 10^{38} \text{ erg/s for } E > 1 \text{ PeV ,} \end{aligned}$$

- Thus, even few powerful galactic point sources could be important
- **Question:** does an energy flux of 10^{38} erg/sec be a small or large quantity on the astrophysical point of view?
- **(Remark:** *this is the fundamental question for any new physical phenomena we meet, i.e., consider the energy of the system and compare with similar quantities/objects*)

Exercise: the CRAB and Tycho SNR

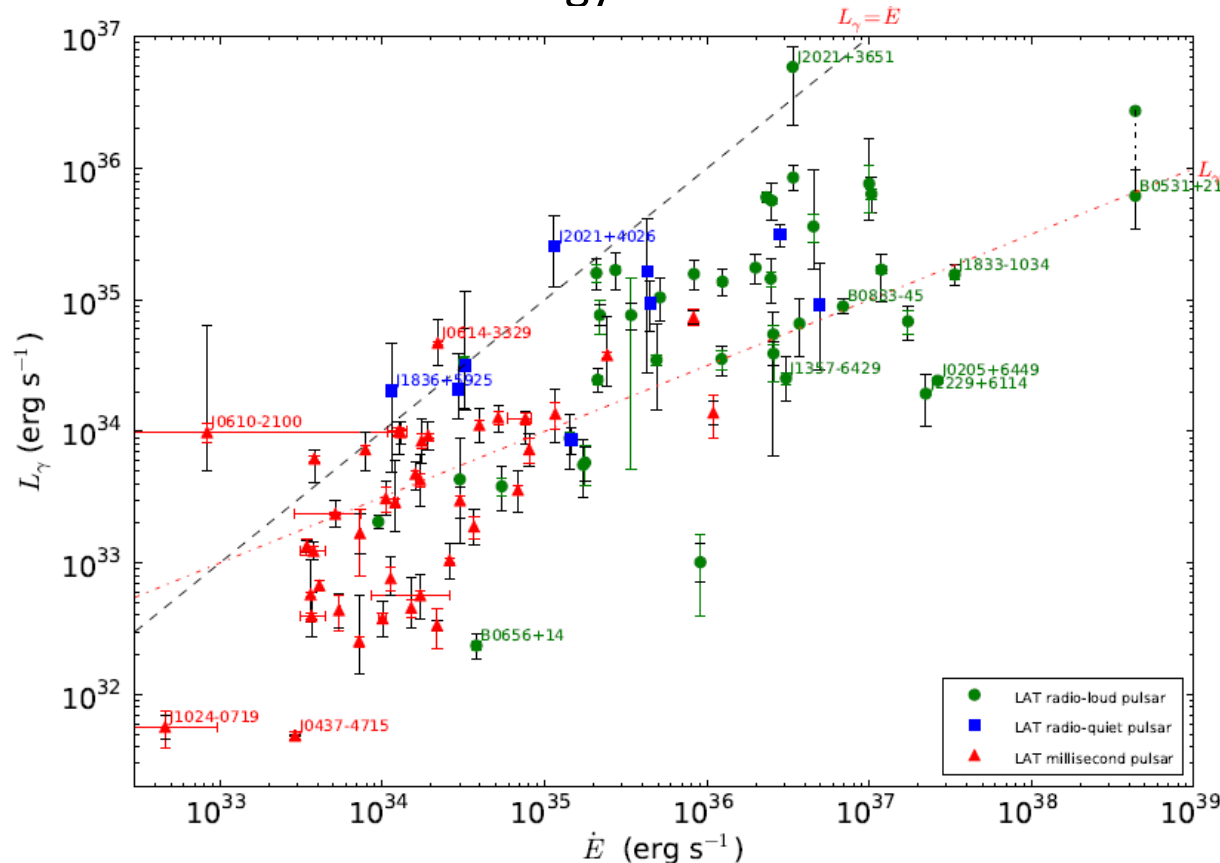
- Distance of the CRAB=2000 pc
- Put on the right y-axis the corresponding emitted energy by the source in units of erg/s. Put in evidence the position of 10^{37} erg/s
- Do the same exercise for the Tycho SNR (2.5 kpc) in Fig. 9.14 of the book
- **Solution for the CRAB** (displace the figure)



Multiwavelength observations of the Crab nebula. Refer to Chap. 8 for details

Luminosity in the gamma-ray band

- Observed luminosity of binary system and pulsars in gamma-rays from the Fermi-LAT vs. energy loss



Exercise:
estimate the
pulsar's lifetime

Fig. 9 from “The Second Fermi Large Area Telescope Catalog of Gamma-ray Pulsars.”
<https://arxiv.org/abs/1305.4385> Astrophysical Journal Supplement 208, 17 (2013)

A Simple Model Involving Pulsars

- High magnetic fields spinning around the nonaligned axis of rotation will produce strong electric fields \mathcal{E} through Faraday's law (6.1), i.e., order of:

$$\frac{\mathcal{E}}{L} = \frac{1}{c} \frac{dB}{dt}$$

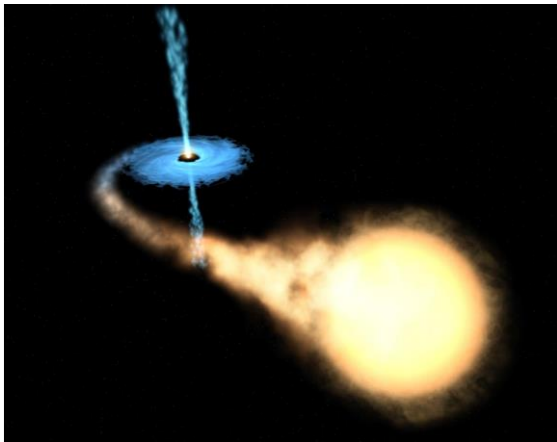
- the maximum energy E_{\max} gained from a particle passing close to a pulsar rotating with angular velocity ω_{NS} over the length $L \sim R_{\text{NS}}$ is

$$E^{\max} = \int Ze\mathcal{E} dx = \int Ze \frac{L}{c} \frac{dB}{dt} dx = \int Ze \frac{L}{c} dB \frac{dx}{dt} = ZeR_{\text{NS}}B \frac{\omega_{\text{NS}}R_{\text{NS}}}{c},$$

- Using the [derived value](#) of the magnetic field and with $(\omega_{\text{NS}}R_{\text{NS}}/c) \sim 0.1$

$$E^{\max} = ZeR_{\text{NS}}B \frac{\omega_{\text{NS}}R_{\text{NS}}}{c} = 4.8 \times 10^{-10} [\text{statC}] \times 10^6 [\text{cm}] \times 10^{11} [\text{Gauss}] \times 0.1$$

$$\simeq 5 \times 10^6 \text{ erg} \simeq 3 \times 10^{18} \text{ eV},$$



- Thus, some galactic accelerators could explain the presence of CRs with energies up to $\sim 10^{18} - 10^{19}$ eV.
- A similar model holds for **microquasars** (binary systems, see § 6.9.2)

Which model for $10^{15} < E_{CR} < 10^{19}$ eV is the correct one?

