Saturday, September 17, 2022 11:59 AM

CALCOLO DIFFERENZIALE

2022/23

LEZIONE 1

SPAZI METRICI

$$(X, d)$$

INSIEME (X) — METRICA (DISTANZA) (d) CIOÈ:

$$d : X \times X \longrightarrow \mathbb{R}$$

tale che

i) $d(x, x') \geq 0 \quad \forall x, x' \in X$ ($\forall$: PER OGNI)

$+$

$$d(x, x') = 0 \iff x = x'$$

ii) $d(x, x') = d(x', x)$ SIMMETRIA

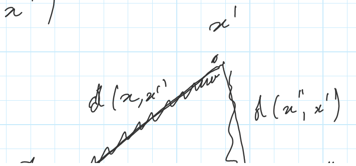
iii) $\forall x, x', x''$ SI HA:

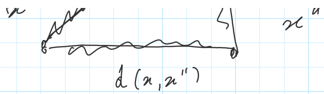
$$d(x, x') \leq d(x, x'') + d(x'', x')$$

DISUGUAGLIANZA TRIANGOLARE

PERCHÈ?

$d(x, x')$



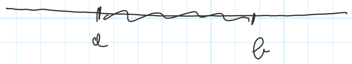


ESEMPI

1) $\mathbb{R}$ EUCLIDEO OVE

$$d(a, b) \overset{DEF}{=} |b - a| \qquad a, b \in \mathbb{R}$$

$$|b - a| = |a - b| \qquad \text{SIMMETRIA}$$



2) $\mathbb{R}^n$ EUCLIDEO, $n \in \mathbb{Z}^+$

$$\underline{x} = (x_1, x_2, \dots, x_n) \quad , x_i \in \mathbb{R}$$

$$\underline{y} = (y_1, y_2, \dots, y_n) \quad , y_i \in \mathbb{R}$$

SI PONE:

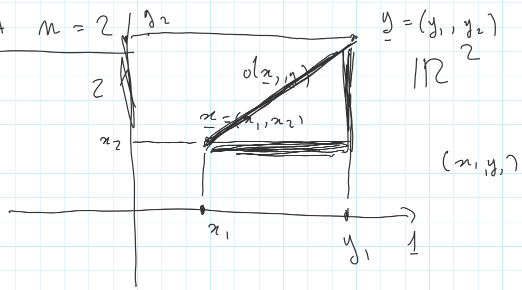
$$d(\underline{x}, \underline{y}) \overset{DEF}{=} \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + \dots + (y_n - x_n)^2} \quad (*)$$

METRICA (!!!) EUCLIDEA.

BREAK INIZIO ORE 15.10?

COSA SIGNIFICA $(*)$?

SIA $n = 2$



(*) DIVENTA:

$$d(\underline{x},\underline{y}) = \sqrt{(y_1-x_1)^2+(y_2-x_2)^2}$$

$(y_1-x_1)^2$ = lunghezza di $(x_1,y_1)^2$

$(y_2-x_2)^2$ = lunghezza di $(x_2,y_2)^2$

<u>TEOREMA DI PITAGORA!!!</u>

1) <u>INTORNO SFERICO APERTO</u>

(X, d) <u>SPAZIO METRICO</u>

SIANO $\underline{x} \in X$ (CENTRO)

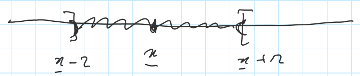
& $r \in \mathbb{R}^+$ (RAGGIO)

$$I(\underline{x}, r) \overset{DEF}{=} \{x \in X \,;\, d(x,\underline{x}) < r\}$$

CENTRO RAGGIO

<u>ESEMPI</u> 1) $\mathbb{R}$ EUCLIDEO

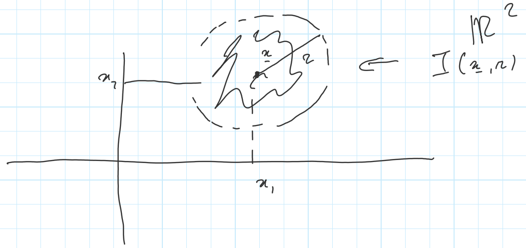
$$I(\underline{x}, r) \overset{DEF}{=} \{x \in \mathbb{R} \,;\, |x-\underline{x}| < r\} \quad r \in \mathbb{R}^+$$



$$\{x \in \mathbb{R} \,;\, \underline{x}-r < x < \underline{x}+r\}$$

2) $\mathbb{R}^2$ EUCLIDEO

$\underline{x} = (x_1, x_2)$ $r \in \mathbb{R}^+$



2) <u>INSIEME APERTO</u>

(X, d) <u>SP. METRICO</u>

$A \subseteq X$

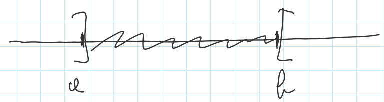
A <u>APERTO</u> $\overset{DEF}{\iff}$ $\forall x \in A \ \exists \underset{\text{RAGGIO}}{r} \in \mathbb{R}^+$ t.c.

$$I(x, r) \subseteq A$$

<u>ESEMPIO IN $\mathbb{R}$ EUCLIDEO</u> $\quad a, b \in \mathbb{R}$, $a < b$

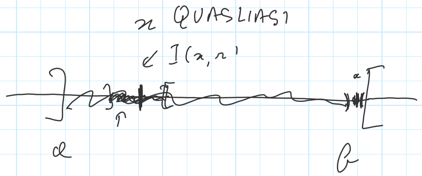
E SIA $\ ]a, b[\overset{DEF}{=} \{ x \in \mathbb{R} \ ; \ a < x < b \}$

CIOÈ



ORA $\ ]a, b[\ $ È APERTO !!!

INFATTI,



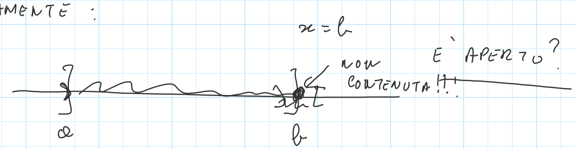
BASTA CHE SIA

$$r \leq d(x, a), d(x, b)$$

<u>CONTROESEMPIO</u>

SIA: $\ ]a, b] \overset{DEF}{=} \{ x \in \mathbb{R} \ ; a < x \leq b \}$

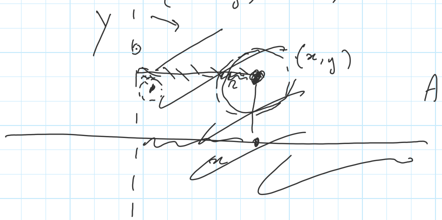
GRAFICAMENTE:



NO PERCHÉ LA CONDIZIONE DI ESSERE APERTO FALLISCE PER $x = b$!!!

ES 3 $\mathbb{R}^2$ EUCLIDEO.

SIA $A = \{(x,y) \in \mathbb{R}^2 \; ; \; x > 0\}$ CIOÈ



$\mathbb{R}^2$

A APERTO?

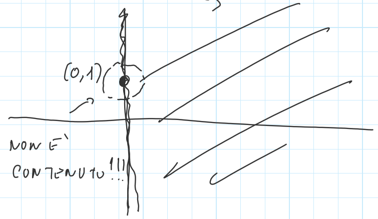
$I((x,y), r) \subseteq A$ SE

$r \leq d((x,y)$, ASSE DELLE $y) =$

$= x > 0$

CONTROESEMPIO $\mathbb{R}^2$ EUCLIDEO

$B = \{(x,y) \in \mathbb{R}^2 \; ; \; 0 \leq x\}$ CIOÈ



QUINDI, SE PRENDO $(0,1) \in B$!!!

$\nexists r \in \mathbb{R}^+$, $I((0,1), r)) \not\subseteq B$!!!

QUINDI, LA COND. DI APERTURA È FALSA!

$\Longrightarrow$ B NON APERTO !!!

DOMANDE?

BYE