Thursday, September 22, 2022 10:57 AM

INIZIO ORE 14.10

$\mathbb{R}^m$ SP. VETTORIALE !!!

DATI $\underline{x} = (x_1, x_2, \dots, x_m)$, $x_i \in \mathbb{R}$

$\underline{y} = (y_1, y_2, \dots, y_m)$, $y_i \in \mathbb{R}$

+) SOMMA

$\underline{x} + \underline{y} = (x_1+y_1, x_2+y_2, \dots, x_m+y_m)$

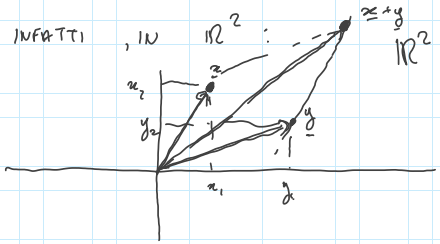
·) $\lambda \in \mathbb{R}$

$\lambda \underline{x} = (\lambda x_1, \lambda x_2, \dots, \lambda x_m)$

VI È UN VETTORE "ZERO" ("ORIGINE")

$\underline{0} = (0,0,0,\dots,0) \in \mathbb{R}^m$

INFATTI, IN $\mathbb{R}^2$:

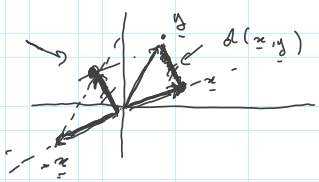


$\mathbb{R}^2$

CHI È $\underline{x}+\underline{y}$???

LE STRUTTURE DI "SP. METRICO" & "SP. VETTORIALE" SONO COLLEGATE ???

CHI È $\underline{y}-\underline{x}$? → " $\underline{y}+(-\underline{x})$



CHI È $\underline{y}-\underline{x}$?

SPAZI NORMATI

$(X, \|\ \|)$ OVE $\|\ \| : X \longrightarrow \mathbb{R}$ NORMA

↑ NORMA

DEF.

SP. VETTORIALI

$x \in X$

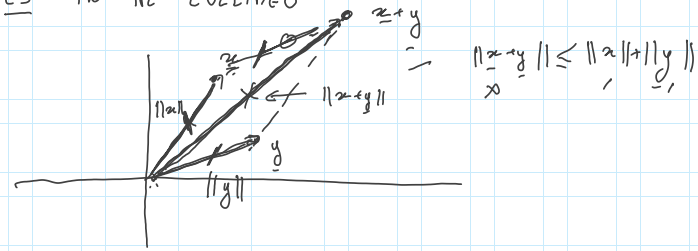
i) $\|x\| \geq 0$ & $\|x\| = 0 \in \mathbb{R} \iff x = \underline{0} \in X$

ii) $\lambda \in \mathbb{R}$ $\|\lambda \cdot x\| = |\lambda| \cdot \|x\|$

iii) $\|x + y\| \leq \|x\| + \|y\|$, $x, y \in X$ SP. VETTORIALE.

(DISUGUAGLIANZA TRIANGOLARE PER LE NORME)

ES IN $\mathbb{R}^2$ EUCLIDEO



$\|\underset{0}{x} + \underset{0}{y}\| \leq \|x\| + \|y\|$

RMK SI HA CHE, POSTO,

$d : X \times X \longrightarrow \mathbb{R}$ t.c. $x, y \in X$

$d(x, y) \overset{DEF}{=} \|x - y\| <$ (*)

È UNA METRICA!

RMK (*) $\Longrightarrow \|x\| = d(x, \underline{0})$

CASO PART.

$\mathbb{R}^n$ NORMATO EUCLIDEO $\iff x = (x_1, \dots, x_n)$

$$\|x\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$ NORMA EUCLIDEA

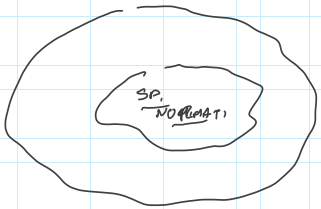
DATA $\|x\|$ NORMA EUCLIDEA (?) SI HA

$d(x, y) \overset{DEF}{=} \|x - y\| =$

↑ EUCLIDEA

↑ EUCLIDEA

$$= \sqrt{(x_1-y_1)^2+(x_2-y_2)^2+\dots+(x_n-y_n)^2}$$



SP. METRICI

SOTTOSPAZI ???

$(X, \|\ \|)$ SP. NORMATO & SIA $\emptyset \neq A \subseteq X$.

SP. VETTORIALE

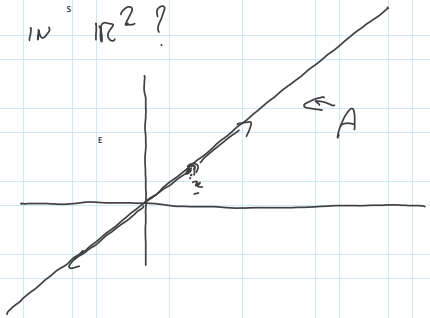
IN QUALI CASI A "EREDITA" UNA STRUTTURA DI SPAZIO NORMATO ???

DOVRA' ESSERE CHE ANCHE A E' SPAZIO VETTORIALE, CIOE' A SOTTOSPAZIO VETTORIALE, CIOE'

i) $\underline{x}, \underline{y} \in A \Rightarrow \underline{x}+\underline{y} \in A$

ii) $\lambda \in \mathbb{R}$, $\Rightarrow (\underline{x} \in A \Rightarrow \lambda \underline{x} \in A)$

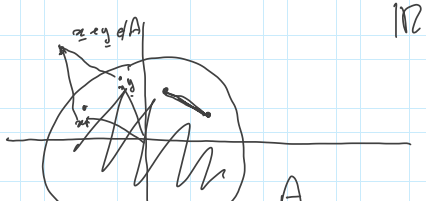
IN $\mathbb{R}^2$?



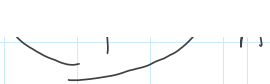
$\mathbb{R}^2$

SI' PERCHE' SOTTOSPAZIO!

VICEVERSA, SIA



$\mathbb{R}^2$



BREAK DOMANDE ?

INIZIO ORE 15.10