SP. CON PROD. INTERNO $(X, \langle , \rangle)$

CIOÈ $\langle , \rangle : X \times X \to \mathbb{R}$

È BILINEARE, SIMMETRICA, DEFINITA POSITIVA.

IN $\mathbb{R}^n$, SI HA PROD. INTERNO EUCLIDEO :

$\underline{x} = (x_1, \dots, x_n)$, $\underline{y} = (y_1, \dots, y_n) \in \mathbb{R}^n$

$$\langle \underline{x}, \underline{y} \rangle = x_1 y_1 + x_2 y_2 + \dots + x_n y_n = \sum_{i=1}^{n} x_i y_i \quad (*)$$

SYLVESTER'S "LAW OF INERTIA"

IL PROD. INTERNO EUCLIDEO È "ESSENZIALMENTE" (???)

L'UNICO PRODOTTO INTERNO IN $\mathbb{R}^n$.

RICORDIAMO CHE:

$$\|x\| \stackrel{DEF}{=} \sqrt{\langle x, x \rangle}$$ NORMA !!!

DI PIÙ

$$d(\underline{x}, \underline{y}) \stackrel{DEF}{=} \|x - y\| = \sqrt{\langle x-y, x-y \rangle}$$ METRICA !!!

AUGURI PREMESSA

DISUGUAGLIANZA DI CAUCHY / SCHWARTZ

$(X, \langle , \rangle)$ SP. CON PROD. INTERNO. SI HA

THM DATI $\underline{x}, \underline{y} \in X$

$$(\dagger) \quad |\langle \underline{x}, \underline{y} \rangle| \le \sqrt{\langle \underline{x}, \underline{x} \rangle} \sqrt{\langle \underline{y}, \underline{y} \rangle} = \|\underline{x}\| \|\underline{y}\| \quad !!!$$

ORA, SIANO $\underline{x}, \underline{y} \neq 0$, PERCIÒ LA $(\dagger) \Rightarrow$

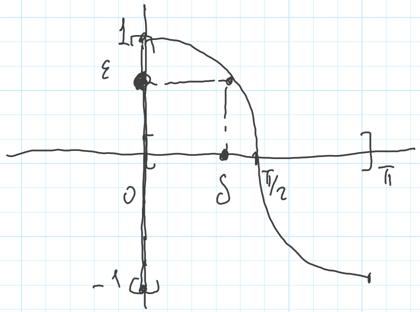
$$(\ddagger) \quad \frac{|\langle x, y \rangle|}{\|x\| \cdot \|y\|} \le 1$$

$$\Updownarrow$$

$$(\#) \quad -1 \le \frac{\langle \underline{x}, \underline{y} \rangle}{\|\underline{x}\| \cdot \|\underline{y}\|} \le 1$$

(IN $\mathbb{R}^n$)

$$\cos : [0, \pi] \longrightarrow \mathbb{R}$$



$$\cos : [0, \pi] \xrightarrow[1-1]{su} [-1, 1]$$

$$\Updownarrow$$

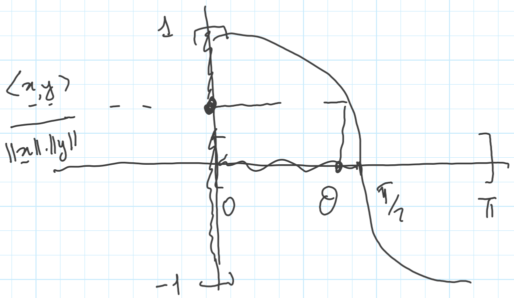
INVERTIBILE !!!

DATO $\varepsilon \in [-1, 1]$ $\exists! \, \delta \in [0, \pi]$

t.c. $\cos \delta = \varepsilon$.

MA ORA (RICORDO):

$$(\#) \quad -1 \le \frac{\langle \underline{x}, \underline{y} \rangle}{\|x\| \cdot \|y\|} \le 1$$



$\exists!\ \theta \in [0, \pi]$

t.c.

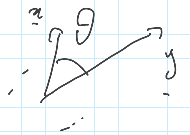
$$\cos \theta = \frac{\langle \underline{x}, \underline{y} \rangle}{\|\underline{x}\| \|\underline{y}\|}$$

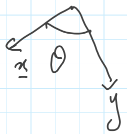
<u>PER DEFINIZIONE</u>, $\theta \in \pi$ È <u>L'ANGOLO</u>

TRA I VETTORI (<u>NON ZERO</u>) $\underline{x}, \underline{y} \in \mathbb{R}^n$ $(\underline{x}, \underline{y} \neq \underline{0})$

ANGOLO NON ORIENTATO POICHÉ IL PROD. INTERNO

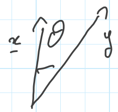
È SIMMETRICO $(\Leftrightarrow \langle \underline{x}, \underline{y} \rangle = \langle \underline{y}, \underline{x} \rangle)$



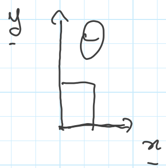


<u>ORA</u> $\underline{x}, \underline{y} \neq 0$

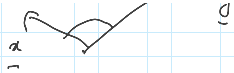
1) $\langle \underline{x}, \underline{y} \rangle > 0 \Leftrightarrow 0 \leq \theta < \pi/2$, θ È <u>ACUTO</u>!!!

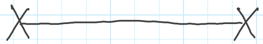


2) $\langle \underline{x}, \underline{y} \rangle = 0 \Leftrightarrow \theta = \frac{\pi}{2}$, θ È <u>RETTO</u> $\Leftrightarrow$ $\underline{x}, \underline{y}$ SONO <u>ORTOGONALI</u>



3) $\langle \underline{x}, \underline{y} \rangle < 0 \Leftrightarrow \frac{\pi}{2} < \theta \leq \pi$, θ È <u>OTTUSO</u>.





BREAK DOMANDE?

INIZIO 12.05