Friday, September 30, 2022 11:02 AM

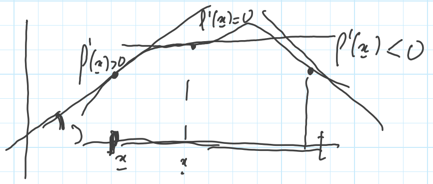
CALCOLO DIFFERENZIALE

RICORDO IN $\mathbb{R}$

$f : A \subseteq \mathbb{R} \to \mathbb{R}$, A APERTO , $\underline{x} \in A$.

f È DERIVABILE IN $\underline{x} \in A \overset{DEF}{\iff} \exists$ FINITO

$$\lim_{t \to 0} \frac{f(\underline{x}+t) - f(\underline{x})}{t} \overset{DEF}{=} f'(\underline{x}).$$



FATTO FONDAMENTALE

f È DERIVABILE IN $\underline{x}$ $\Longrightarrow$ f È CONTINUA IN $\underline{x}$.

$\not\Longleftarrow$

IN $\mathbb{R}^n$???

DIREZIONE (VERSORE) IN $\mathbb{R}^n$

$v \in \mathbb{R}^n$ DIREZIONE $\iff \|v\| = 1$

EX 1) IN $\mathbb{R}$ $(n=1)$ CHI SONO I VERSORI.

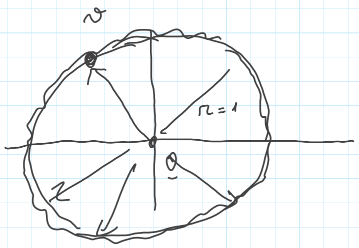
$x \in \mathbb{R}$, $\|x\| = |x| = 1 \Rightarrow \left(x \text{ VERSORE} \iff x = \begin{smallmatrix} -1 \\ 1 \end{smallmatrix} \right)$

-1 0 1 $\mathbb{R}$

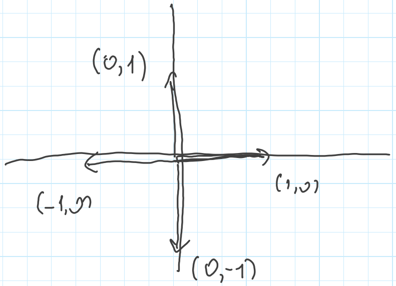
EX 2 IN $\mathbb{R}^2$, CHI SONO I VERSORI?

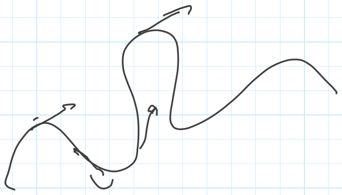
$v=(v_1,v_2)\in\mathbb{R}^2$, v VERSORE $\overset{\text{def}}{\iff}$

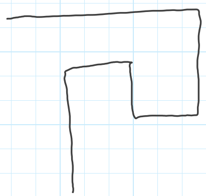
$d(v,\underline{0}) = \|v\| = 1$



$\mathbb{R}^2$





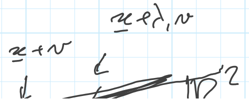


$f : A \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$, A APERTO , $\underline{x}\in A$.

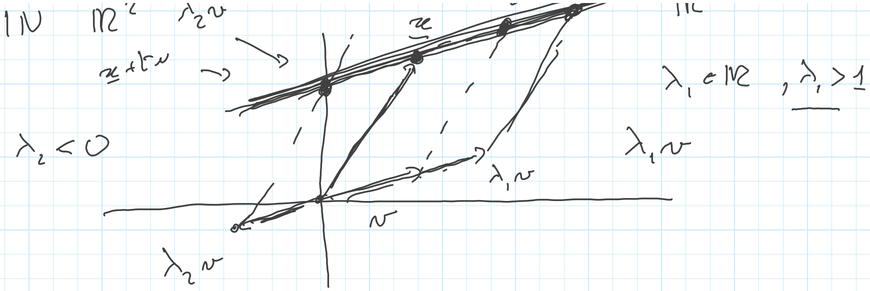
NOTIAMO CHE DATO $\underline{x}\in\mathbb{R}^n$, v versore SIA

$$r_{\underline{x},v} = \{\underline{x}+tv \ ; \ t\in\mathbb{R}\} \subseteq \mathbb{R}^n$$

CHI È $r_{\underline{x},v}$???



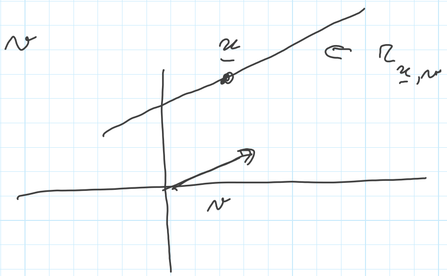
IN $\mathbb{R}^2$



$r_{\underline{x},v} \overset{DEF}{=} \{\underline{x}+tv;\ t\in\mathbb{R}\}$ È L' UNICA

RETTA PASSANTE PER $\underline{x}$ ED AVENTE DIREZIONE (PARALLELA)

A v



DERIVATE DIREZIONALI $f: A \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$, A APERTO, $\underline{x}\in A$.

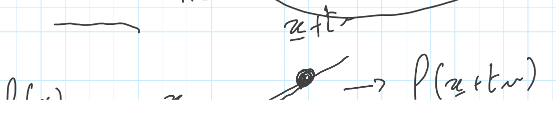
DEF

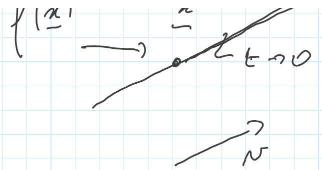
f AMMETTE DER. DIREZIONALE IN $\underline{x}$ SECONDO LA DIREZIONE v

$\Updownarrow$ DEF

$\exists$ FINITO

$$\lim_{t\to 0\in\mathbb{R}} \frac{f(\underline{x}+tv)-f(\underline{x})}{t}$$





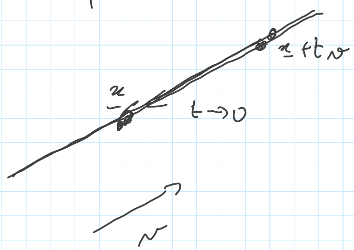
$t \to 0 \iff x + tv \xrightarrow[t \to 0]{} \underline{x}$

<u>NB</u> $\quad H_p \quad f : A \subseteq \mathbb{R}^n \to \mathbb{R} \qquad n > 1$

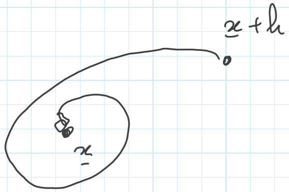
AMMETTE <u>TUTTE</u> LE DERIVATE DIREZIONALI IN $\underline{x}$

$\Downarrow$? <u>NO</u> (È FALSO)

f È CONTINUA IN $\underline{x}$



$\underline{x} + h \xrightarrow[h \to \underline{0}]{} \underline{x}$



IN $\mathbb{R}^2$

$\Rightarrow$