Wednesday, October 5, 2022 9:35 AM

# INIZIO ORE 14.10

## DERIVATE DIREZIONALI

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, $A$ APERTO, $\underline{x} \in A$.

SIA $v : \|v\| = 1$ UNA DIREZIONE.

**DEF** $f$ AMMETTE DERIVATA DIREZIONALE IN $\underline{x} \in A$ SECONDO LA DIREZIONE $v : \|v\| = 1$

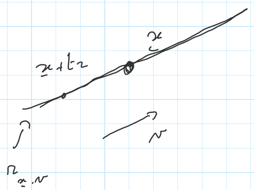
$\overset{DEF}{\iff}$

$$\exists \text{ FINITO } \lim_{t \to 0 \in \mathbb{R}} \frac{f(\underline{x}+tv) - f(\underline{x})}{t} \quad (*)$$

RAPP. INCREMENTALE

OVE $r_{\underline{x},v} = \{\underline{x}+tv;\ t \in \mathbb{R}\}$ L'UNICA RETTA PASSANTE $\underline{x} \in A$ E "PARALLELA" ALLA DIREZIONE $v$

DDD ...




ORA, SE $n = 1$, SIA $v = \underline{1} \in \mathbb{R}$

ALLORA $(*)$ DIVENTA:

$$\lim_{t \to 0} \frac{f(\underline{x}+t) - f(\underline{x})}{t} = f'(\underline{x})$$

DERIVATA ORDINARIA IN UNA VARIABILE

$\to$ RAPPORTO INCREMENTALE.

ORA, IN $n=1$ SI HA:

$f$ AMMETTE DERIVATA $f'(\underline{x})$ IN $\underline{x}$

$\Downarrow$

$f$ CONTINUA IN $\underline{x} \in A$

IN PIÙ VARIABILI? CIOÈ, SE $n > 1$?

È VERO:

$f$ AMMETTA <u>TUTTE</u> LE DER. DIREZIONALI IN $\underline{x}$

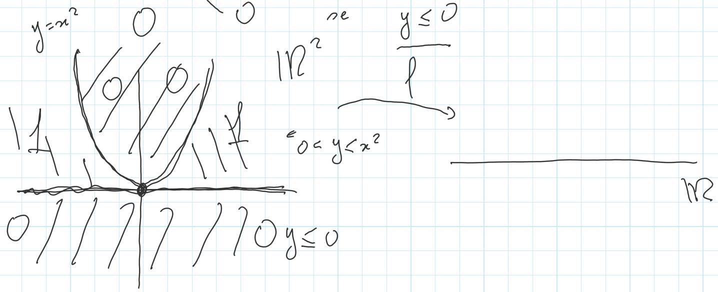
? $\Downarrow$ <u>NO</u> <u>È FALSO</u>

$f$ È CONTINUA IN $\underline{x} \in A$.

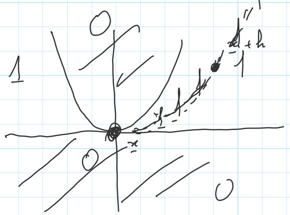
<u>CONTROESEMPIO</u> $n=2$ SIA

$f: \mathbb{R}^2 \to \mathbb{R}$ OVE

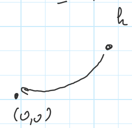
$$f(x,y) = \begin{cases} 0 & \text{se } \underline{y \geq x^2} \\ 1 & \text{se } \underline{0 < y < x^2} \\ 0 & \text{se } \underline{y \leq 0} \end{cases} \qquad (x,y) \in \mathbb{R}^2$$




<u>ORA</u>, SIA $\underline{x} = (0,0) \Rightarrow f(\underline{x}) = f(0,0) = 0$




$h \to \underline{0} \in \mathbb{R}^2$



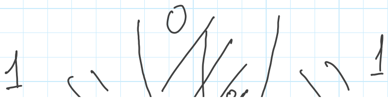

$f(\underline{x}+h) = 1 \quad h \neq \underline{0}$

MA

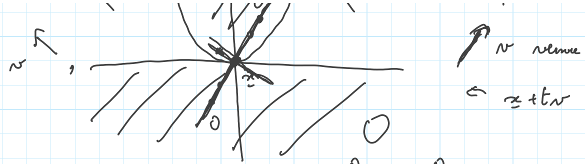
$f(\underline{x}) = 0$ !!!

<u>PERCIÒ</u>, <u>$f$ NON È CONTINUA IN $\underline{x} = (0,0)$ !!!</u>

RIGUARDO ALLE DER. DIREZIONALI IN $\underline{x} = (0,0)$ ???




$f(\underline{x}) = f(0,0) = 0$




QUINDI , $\lim_{t \to 0} \frac{f(\underline{x}+t v) - f(\underline{x})}{t} = 0$ !!!

ABBIAMO DIMOSTRATO CHE , $\forall v : \|v\| = 1$ DIREZIONE

$$\exists \frac{\partial f}{\partial v}(\underline{x}) = 0 \quad !!!$$

## FUNZIONI DIFFERENZIABILI

SIA $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO , $\underline{x} \in A$

### DEF. FONDAMENTALE

$f$ DIFFERENZIABILE IN $\underline{x} \in A$

$\Updownarrow$ DEF

$\exists \; L_{\underline{x}} : \mathbb{R}^n \to \mathbb{R}$ FUNZ. LINEARE t.e.

$$\lim_{\underline{h} \to \underline{0} \in \mathbb{R}^n} \frac{f(\underline{x}+\underline{h}) - f(\underline{x}) - L_{\underline{x}}(\underline{h})}{\|\underline{h}\|} = 0 \quad !!! \quad (*).$$

RICORDO

$$L_{\underline{x}} \text{ LINEARE} \iff \begin{cases} L_{\underline{x}}(h_1 + h_2) = L_{\underline{x}}(h_1) + L_{\underline{x}}(h_2) & \text{ADDITIVITA'} \\ L_{\underline{x}}(\lambda \cdot h) = \lambda \cdot L_{\underline{x}}(h) & \text{OMOGENEITA'} \end{cases}.$$

COSA SIGNIFICA LA COND. $(*)$ ???

SIA $E_{\underline{x}}(\underline{h}) \overset{DEF}{=} f(\underline{x}+\underline{h}) - f(\underline{x}) - L_{\underline{x}}(\underline{h}) =$

$$= f(\underline{x}+\underline{h}) - \Big( \underbrace{f(\underline{x})}_{\text{COST}} + \underbrace{L_{\underline{x}}(\underline{h})}_{\text{VALORE DI FUNZ LINEARE !!!}} \Big)$$

ERRORE CHE SI COMMETTE APPROSSIMANDO ...

ERRORE CHE SI COMMETTE APPROSSIMANDO

IL VALORE $f(\underline{x}+\underline{h})$ IN $\underline{x}+\underline{h}$

CON $\underset{\text{COST}}{f(\underline{x})} + \underset{\text{LINEARE}}{L_{\underline{x}}(\underline{h})}$ !!!

PERCIÒ

$f$ DIFFERENZIABILE IN $\underline{x}$ $\iff$ $\dfrac{E_{\underline{x}}(\underline{h})}{\|\underline{h}\|} \xrightarrow[\underline{h}\to\underline{0}\in\mathbb{R}]{} 0$

SI NOTI $\underline{h}\to\underline{0} \iff \|\underline{h}\| \xrightarrow[\underline{h}\to\underline{0}]{} 0$

$$\Downarrow \quad \not\Uparrow$$

$$E_{\underline{x}}(\underline{h}) \xrightarrow[\underline{h}\to\underline{0}]{} 0$$

ESPERIMENTO

$f: \mathbb{R}^n \to \mathbb{R}$ SIA LINEARE. È VERO

CHE ALLORA $f$ DIFFERENZIABILE IN $\underline{x}$ $\forall \underline{x}\in\mathbb{R}^n$ ?

SE SÌ, CHI SARANNO I DIFFERENZIALI $L_{\underline{x}}$ ???

OVVIAMENTE $L_{\underline{x}} = f \quad \forall \underline{x}\in\mathbb{R}^n$.

INFATTI:

$$\lim_{\underline{h}\to\underline{0}} \frac{f(\underline{x}+\underline{h}) - f(\underline{x}) - L_{\underline{x}}(\underline{h})}{\|\underline{h}\|} =$$

NEL NOSTRO CASO È:

$$= \lim_{\underline{h}\to\underline{0}} \frac{f(\underline{x}+\underline{h}) - f(\underline{x}) - f(\underline{h})}{\|\underline{h}\|} =$$

$f$ È LINEARE $\implies$

$f(\underline{x}+\underline{h}) = f(\underline{x}) + f(\underline{h})$

$$= \lim_{h \to 0} \frac{\cancel{f(x)} + \cancel{f(h)} - \cancel{f(x)} - \cancel{f(h)}}{\|h\|} = 0$$

BREAK DOMANDE?

INIZIO 15.15