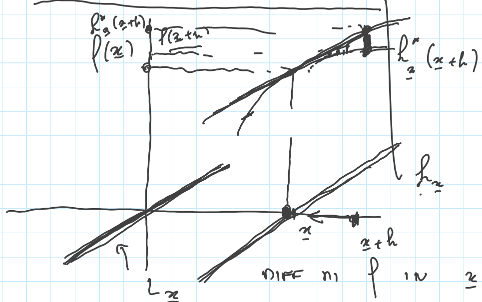
Wednesday, October 5, 2022 9:36 AM

"GRAFICAMENTE" IN $n=1$



$f:\mathbb{R}\to\mathbb{R}$

DIFF DI f IN $x \in \mathbb{R}$

LINEARE

SIA $\mathcal{L}_x(x+h) = L_x(h)$ SIA

$$\mathcal{L}^*_x(x+h) = \mathcal{L}_x(x+h) + f(x)$$ COST

$$f(x+h) - \mathcal{L}^*_x(x+h) = f(x+h) - f(x) + \mathcal{L}_x(x+h) \overset{DEF}{=}$$

$$= f(x+h) - f(x) - L_x(h) = E_x(h)$$

PROP SE $n=1$

f AMMETTE DERIVATA $f'(x)$ IN x

$$\Updownarrow$$

f DIFFERENZIABILE IN x

PROOF $(\Leftarrow)$ f DIFFERENZIABILE IN x $\overset{DEF}{\Longleftrightarrow}$

$\exists\, L_x : \mathbb{R} \to \mathbb{R}$ LINEARE t.c.

$$\lim_{h\to 0\in\mathbb{R}} \frac{f(x+h)-f(x)-L_x(h)}{|h|} = 0 \qquad (*)$$

$? \Updownarrow$ SÌ

l. $f(x+h) - f(x) - L_x(h)$

$$\lim_{h\to 0\in\mathbb{R}} \frac{\qquad}{h} = 0 \quad (**)$$

$$(**) \iff \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{L_x(h)}{h}$$

L_x LINEARE

$$= \lim_{h\to 0} \frac{L_x(h\cdot 1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\cdot \overbrace{L_x(1)}^{\text{COST}}}{\cancel{h}} = L_x(1) \text{ COST.}$$

PERCIÒ $\exists\ f'(x) = L_x(1)$

ABBIAMO PROVATO:

f DIFF $\Rightarrow f$ HA DERIVATA

E DI PIÙ

$f'(x) = L_x(1)$

ORA $\Rightarrow$) SIA $L_x(h) \overset{\text{DEF}}{=} \overbrace{f'(x)}^{\text{COST}}\cdot h$

È VERO CHE:

$$\lim_{h\to 0\in\mathbb{R}} \frac{f(x+h)-f(x)-f'(x)\cdot h}{|h|} = 0 \ ? \quad (\dagger)$$

$$\Updownarrow$$

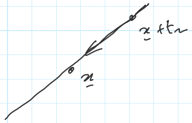
$$\lim_{h\to 0\in\mathbb{R}} \frac{f(x+h)-f(x)-f'(x)\cdot h}{h} = 0 \ ? \quad (\dagger\dagger)$$

$$\Updownarrow$$

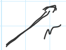
$$\lim_{h \to 0 \in \mathbb{R}} \frac{f(\underline{x}+h) - f(\underline{x})}{h} \overset{!}{=} \lim_{h \to 0} \frac{f'(\underline{x}) \cdot \cancel{h}}{\cancel{h}} = \underline{f'(\underline{x})}$$

VERO PER DEFINIZIONE !!! QED.

$$\exists \underset{\text{FINITO}}{} \frac{\partial f}{\partial v}(\underline{x}) = \lim_{t \to 0} \frac{f(\underline{x}+tv) - f(\underline{x})}{t}$$



$t \to 0$



BREAK DOMANDE