Wednesday, October 5, 2022 9:37 AM

INIZIO ORE 11.10

RICORDO PROP $n=1$ SI HA

f DIFFERENZIABILE IN $\underline{x}$ $\Longleftrightarrow$ f AMMETTE DERIVATA $f'(\underline{x})$ IN $\underline{x}$

DI PIÙ $f'(\underline{x}) = L_{\underline{x}}(1)$ & $L_{\underline{x}}: \mathbb{R} \to \mathbb{R}$ LINEARE

$L_{\underline{x}}(h) = f'(\underline{x}) \cdot h \quad \forall h \in \mathbb{R}.$

PROP. $f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$.

f DIFFERENZIABILE IN $\underline{x}$ $\Longrightarrow$ $\forall v: \|v\|=1$ DIREZIONE

$\exists$ LA DERIVATA DIREZIONALE

$$\frac{\partial f}{\partial v}(x) \overset{\text{TH}}{=} L_{\underline{x}}(v).$$

PROOF. f DIFF IN $\underline{x}$ $\overset{\text{DEF}}{\Longleftrightarrow}$

$\exists L_{\underline{x}}: \mathbb{R}^n \to \mathbb{R}$ LINEARE t.c.

$$\lim_{h \to \underline{0} \in \mathbb{R}^n} \frac{f(\underline{x}+h) - f(\underline{x}) - L_{x}(h)}{\|h\|} = 0 \quad (*)$$

$\Downarrow$ PER SPECIALIZZAZIONE: $h = t \cdot v \quad t \in \mathbb{R}$

$*$ DIVENTA

$$\lim_{tv \to \underline{0}} \frac{f(\underline{x}+tv) - f(\underline{x}) - L_{\underline{x}}(tv)}{\|tv\|} = 0 \quad (**)$$

$\Updownarrow$ $L_{\underline{x}}$ LINEARE

ORA
$\|t \cdot v\| = |t| \|v\| = |t|$

$$\lim_{t \to 0 \in \mathbb{R}} \frac{f(\underline{x}+tv) - f(\underline{x}) - t \cdot L_{\underline{x}}(v)}{|t|} = 0 \quad (***)$$

$\Updownarrow$

$$\lim_{t\to 0} \frac{f(\underline{x}+t\underline{v}) - f(\underline{x}) - t \cdot L_{\underline{x}}(\underline{v})}{t} = 0 \qquad (***)$$

$$\Updownarrow$$

$$(+) \quad \underline{\lim_{t\to 0} \frac{f(\underline{x}+t\underline{v}) - f(\underline{x})}{t}} = \lim_{t\to 0} \frac{\cancel{t} \cdot L_{\underline{x}}(\underline{v})}{\cancel{t}} = L_{\underline{x}}(\underline{v}) \in \mathbb{R}$$

$$(+) \quad \exists \ \frac{\partial f}{\partial \underline{v}}(\underline{x}) \overset{!}{=} L_{\underline{x}}(\underline{v}) \qquad !!! \qquad \underline{QED}$$

<u>PREMESSA</u> <u>BASE CANONICA DI $\mathbb{R}^n$</u> È L'INSIEME

$\{\underline{e}_1, \underline{e}_2, \dots, \underline{e}_n\}$ OVE $i = 1, 2, \dots, n$

$$\underline{e}_i \overset{DEF}{=} (0, 0, \dots, 0, \underset{i\text{-esima}}{1}, 0, \dots, 0)$$

<u>EX $n=3$</u>, $\underline{\underline{e}_1 = (1,0,0), \ \underline{e}_2 = (0,1,0), \ \underline{e}_3 = (0,0,1)}$

DI PIÙ, SI HA:

$$\underline{v} = (v_1, v_2, \dots, v_n) \in \mathbb{R}^n$$

$$\overset{!}{=} \sum_{i=1}^{n} v_i \cdot \underline{e}_i$$

<u>EX</u> $n=3$ $\quad \underline{v} = (2, -1, 3) =$

$$= 2\underset{\underline{e}_1}{(1,0,0)} - 1 \cdot \underset{\underline{e}_2}{(0,1,0)} + 3\underset{\underline{e}_3}{(0,0,1)}$$

DERIVATE PARZIALI $\quad f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$

PER DEFINIZIONE, LE i-ESIMA $(i = 1, 2, \dots, n)$ DERIVATA PARZIALE

È LA SPECIALE DERIVATA DIREZIONALE:

$$\frac{\partial f}{\partial \underline{e}_i}(\underline{x})$$

ORA, DUE DOMANDE

1) PERCHÈ "PARZIALE" ???

2) $\frac{\partial f}{\partial \underline{e}_i}(\underline{x}) \stackrel{?}{=} \frac{\partial f}{\partial x_i}(\underline{x})$???

DEF

SIA $i = 1, 2, \dots, n$ $\quad \underline{x} = (x_1, x_2, \dots, x_i, \dots, x_n) \in \mathbb{R}^n$

CHE $\quad \underline{x} + t\,\underline{e}_i = (x_1, \dots, x_{i-1}, x_i + t, x_{i+1}, \dots x_n)$.

ORE, PER DEFINIZIONE,

$$\frac{\partial f}{\partial \underline{e}_i}(\underline{x}) = \overset{\text{FINITO}}{\lim_{t \to 0}} \frac{f(\underline{x} + t\,\underline{e}_i) - f(\underline{x})}{t} =$$

$$= \lim_{t \to 0} \frac{f(x_1, \dots, x_{i-1}, x_i + t, x_{i+1}, \dots, x_n) - f(x_1, x_2, \dots x_i, \dots x_n)}{t} \quad ???$$

EX $\quad f:\mathbb{R}^2\to\mathbb{R}\ , \ f(x,y) = e^{x^2y} + xy^2$

$$\frac{\partial f}{\partial x}(x,y) = 2xy\cdot e^{x^2y} + y^2$$

$$\frac{\partial f}{\partial y}(x,y) = x^2 e^{x^2y} + 2xy$$

$L_{\underline{x}}:\mathbb{R}^n\to\mathbb{R}$ LINEARE $\quad$ UN DIFFERENZIALE DI UNA FUNZIONE f IN $\underline{x}$.

DATO $v=(v_1,v_2,\ldots,v_n)\in\mathbb{R}^n$ $\quad$ COME SI CALCOLA $L_{\underline{x}}(v)$???

$$\stackrel{!}{=}\sum_{i=1}^{n} v_i\cdot \underline{e}_i$$

PERCIÒ $\quad L_{\underline{x}}(v) = L_{\underline{x}}\left(\sum_{i=1}^{n} v_i\cdot\underline{e}_i\right) \quad \Leftarrow \quad L_{\underline{x}}$ LINEARE !!

$$= \sum_{i=1}^{n} v_i\cdot L_{\underline{x}}(\underline{e}_i)$$

$$= \sum_{i=1}^{n} v_i\cdot\frac{\partial f}{\partial \underline{e}_i}(\underline{x}) \quad \Leftarrow \underline{\text{THM}}$$

$$\longrightarrow = \sum_{i=1}^{n} v_i\cdot\frac{\partial f}{\partial x_i}(\underline{x}) =$$

$$= \left\langle (v_1, v_2, \dots, v_n), \left(\frac{\partial f}{\partial x_1}(\underline{x}), \frac{\partial f}{\partial x_2}(\underline{x}), \dots, \frac{\partial f}{\partial x_n}(\underline{x}) \right) \right\rangle \quad (\circledast)$$

ORA $\quad \text{grad} f(\underline{x}) \overset{\text{DEF}}{=} \left(\frac{\partial f}{\partial x_1}(\underline{x}), \dots, \frac{\partial f}{\partial x_n}(\underline{x}) \right) \in \mathbb{R}^n$

$$(\circledast) = \langle v, \text{grad} f(\underline{x}) \rangle = \langle \text{grad} f(\underline{x}), v \rangle \overset{\text{THM}}{=} L_{\underline{x}}(v)$$

IN PARTICOLARE, SE $v : \|v\| = 1$ (v DIREZIONE)

$$\frac{\partial f}{\partial v}(\underline{x}) = L_{\underline{x}}(v) \overset{!}{=} \langle \text{grad} f(\underline{x}), v \rangle$$

BREAK : DOMANDE

INIZIO 12.15