Wednesday, October 5, 2022 9:37 AM

EX $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = 3x^2y + xy^3$ DIFFERENZIABILE? SI' (TRA POCO)

$\underline{x} = (1,1)$, $v = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ DIREZIONE.

SI CALCOLI

$$\frac{\partial f}{\partial v}(\underline{x}) \qquad !!!$$

$$\frac{\partial f}{\partial x}(x,y) = 6xy + y^3 \Big/_{(1,1)} = 6+1 = 7$$

$$\frac{\partial f}{\partial y}(x,y) = 3x^2 + 3xy^2 \Big/_{(1,1)} = 3+3 = 6$$

PERCIÒ $\operatorname{grad} f(\underline{x}) = (7, 6)$
$\underline{x} = (1,1)$

$\underline{\text{THM}} \Rightarrow \frac{\partial f}{\partial v}(\underline{x}) = \left\langle (7,6), \left(1/2, \frac{\sqrt{3}}{2}\right)\right\rangle = 7/2 + 3\sqrt{3}$ ☒

$\underline{\text{THM}}$ $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO , $\underline{x} \in A$.

f DIFFERENZIABILE IN $\underline{x}$ $\Longrightarrow$ f CONTINUA IN $\underline{x}$

$\underline{\text{PROOF}}$ $\underline{\text{THESIS}}$ f CONTINUA IN $\underline{x}$ $\Longleftrightarrow$ $|f(\underline{x}+h) - f(\underline{x})| \xrightarrow[h \to \underline{0} \in \mathbb{R}^n]{} 0$

$\underline{\text{ORA HP}}$: f DIFFERENZIABILE IN $\underline{x}$ $\overset{\text{DEF}}{\Longleftrightarrow}$

$\exists L_{\underline{x}} : \mathbb{R}^n \to \mathbb{R}$ LINEARE t.c.

$$\lim_{h \to \underline{0}} \frac{f(\underline{x}+h) - f(\underline{x}) - L_{\underline{x}}(h)}{\|h\|} = 0 \qquad (*)$$

(+)

ORA , SIA $\boxed{E_{\underline{x}}(h) = f(\underline{x}+h) - f(\underline{x}) - L_{\underline{x}}(h)}$, E QUINDI LA $(*)$

SI SCRIVE $\frac{E_{\underline{x}}(h)}{\|h\|} \xrightarrow[h \to \underline{0}]{} 0 \Longrightarrow E_{\underline{x}}(h) \xrightarrow[h \to \underline{0}]{} 0$
$\not\Leftarrow$

MA POSSIAMO RISCRIVERE LA (+)

$$f(\underline{x}+h)-f(\underline{x}) = E_{\underline{x}}(h) + L_{\underline{x}}(h)$$

$$\Downarrow \qquad E_{\underline{x}}(h) \xrightarrow[h\to 0]{} 0$$

$$0 \leq |f(\underline{x}+h)-f(\underline{x})| \leq |E_{\underline{x}}(h)| + |L_{\underline{x}}(h)|$$

MA, PER HP DI DIFF, $E_{\underline{x}}(h) \xrightarrow[h\to 0]{} 0 \iff |E_{\underline{x}}(h)| \xrightarrow[h\to 0]{} 0$

ORA

$$|L_{\underline{x}}(h)| = |\langle \text{grad} f(\underline{x}), h \rangle| \leq$$

CAUCHY/SCHWARTZ INPLICA

$$\leq \|\text{grad} f(\underline{x})\| \cdot \|h\|$$

QUINDI: $\underset{\substack{\downarrow h\to 0 \\ 0}}{|L_{\underline{x}}(h)|} \leq \underset{\text{COST}}{\|\text{grad} f(\underline{x})\|} \cdot \underset{\substack{\downarrow h\to 0 \\ 0}}{\|h\|}$

PERCIÒ

$$0 \leq \underset{\substack{\downarrow h\to 0 \\ 0}}{|f(\underline{x}+h)-f(\underline{x})|} \leq \underset{\substack{\downarrow h\to 0 \\ 0}}{|E_{\underline{x}}(h)|} + \underset{\substack{\downarrow h\to 0 \\ 0}}{|L_{\underline{x}}(h)|}$$

THESIS

PERCIÒ f È CONTINUA IN $\underline{x}$!!! QED

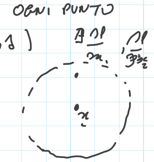
THM (TEOREMA DEL DIFFERENZIALE TOTALE)

$f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$.

HP1 $\exists\ I(\underline{x}, \delta)$ INT. SFERICO APERTO DI $\underline{x}$

PER CUI TUTTE LE DERIVATE PARZIALI

$\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}$ ESISTANO IN OGNI PUNTO DI $I(\underline{x}, \delta)$



HP2 LE $\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}$

"RIGUARDATE" COME FUNZIONI SULL'INTORNO

SIANO <u>CONTINUE</u> IN $\underline{x}$

ALLORA

TH) f È DIFFERENZIABILE IN $\underline{x}$.

<u>CONTROESEMPIO</u> $f: \mathbb{R}^2 \to \mathbb{R}$ OVE

$$f(x,y) = \begin{cases} 2 & xy = 0 \\ 1 & xy \neq 0 \end{cases}$$