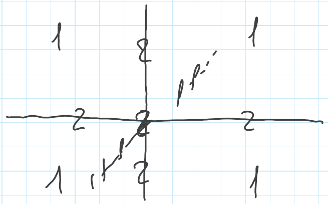
Wednesday, October 5, 2022 12:18 PM

INIZIO ORE 14.10

CONTROESEMPIO $\quad f: \mathbb{R}^2 \to \mathbb{R}$,

$$f(x,y) \begin{cases} 2 & xy = 0 \\ 1 & xy \neq 0 \end{cases} \qquad (x,y) \in \mathbb{R}^2$$



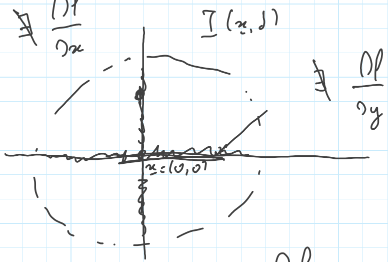
$\underline{x} = (0,0)$

$f(\underline{x}) = f(0,0) = 2$

f NON È CONT IN $\underline{x} \Longrightarrow$

f NON DIFF. IN $\underline{x}$???

$\nexists \dfrac{\partial f}{\partial x}$



$\nexists \dfrac{\partial f}{\partial y}$

TUTTAVIA $\quad \exists \dfrac{\partial f}{\partial x}(\underline{x}) = \dfrac{\partial f}{\partial y}(\underline{x}) = 0$??? $\quad (*)$

$v : \|v\| = 1$

PENSIAMO A: $\quad \dfrac{\partial f}{\partial v}(\underline{x}) = L_{\underline{x}}(v) = \langle \operatorname{grad} f(\underline{x}), v \rangle . \; (\dagger)$

$(*) \Longrightarrow \quad \operatorname{grad} f(\underline{x}) = (0,0)$.

DICO , PRESO $\quad v = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2} \right)$, SI HA

$\underline{x} = (0,0)$

$$\frac{\partial f}{\partial v}(0,0) = \langle \operatorname{grad} f(\underline{x}), v \rangle = \left\langle (0,0), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \right\rangle = 0 \ ???$$

È VERO??? $\quad$ NO!!!

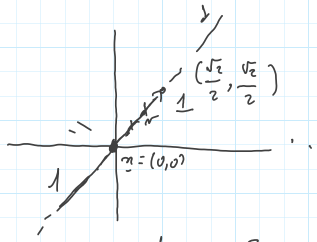
$\underline{x} = (0,0)$

INFATTI $\quad \exists ? \; \dfrac{\partial f}{\partial v}(\;\;) \quad v = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2} \right)$

Infatti, $\exists \; \frac{\partial f}{\partial v}(\underline{x})$ $\quad v = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

cioè

$\exists$ finito $\lim_{t \to 0} \frac{f(\underline{x}+tv) - f(\underline{x})}{t}$?



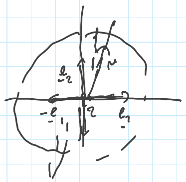
Ma ora:

$$\lim_{t \to 0^+} \frac{\overset{1}{f(\underline{x}+tv)} - \overset{2}{f(\underline{x})}}{t} = \lim_{t \to 0^+} \frac{1-2}{t} = -\infty$$

$$\neq$$

$$\lim_{t \to 0^-} \frac{\overset{1}{f(\underline{x}+tv)} - \overset{2}{f(\underline{x})}}{t} = \lim_{t \to 0^-} \frac{1-2}{t} = +\infty$$

NB Questa funzione in $\underline{x} = (0,0)$



ammette derivate direzionali in $\underline{x}$ soltanto

per le direzioni:

$v = \underline{e}_1, \underline{e}_2, -\underline{e}_1, -\underline{e}_2$

Notazione $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A aperto, $\underline{x} \in A$

f differenziabile in $\underline{x}$ e sia

sia $L_{\underline{x}} : \mathbb{R}^n \to \mathbb{R}$ lineare il differenziale

di f in $\underline{x}$.

Scriviamo …

SCRIVEREMO (AL POSTO DI $L_{\underline{x}}$)

$$df(\underline{x}).$$

PROP $f, g : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$

SIANO f, g DIFFERENZIABILI IN $\underline{x}$.

ALLORA:

1) $f+g$ DIFF IN $\underline{x}$. DI PIÙ $d(f+g)(\underline{x}) = df(\underline{x}) + dg(\underline{x})$ 1 BIS)

2) fg DIFF IN $\underline{x}$. DI PIÙ

$\rightsquigarrow d(fg)(\underline{x}) = \underset{\text{FUNZ LIN}}{df(\underline{x})} \cdot \underset{\text{COST}}{g(\underline{x})} + \underset{\text{COST}}{f(\underline{x})} \cdot \underset{\text{FUNZ LIN}}{dg(\underline{x})}$. 2 BIS)

3) $\lambda \in \mathbb{R}$,

(λf) DIFF IN $\underline{x}$.

DI PIÙ $d(\lambda f)(\underline{x}) = \lambda \cdot df(\underline{x})$ 3 BIS) .

$n = 1$ ORA, IN QUESTO CASO

f DIFF IN $\underline{x} \iff f$ DERIVABILE IN x

E DI PIÙ $f'(x) = df(1)$, $g'(\underline{x}) = dg(1)$.

1) $\Rightarrow$ 1 BIS)

$$\begin{array}{ccccc} d(f+g)(\underline{x})(1) & = & df(\underline{x})(1) & + & dg(\underline{x})(1) \\ \| & & \| & & \| \\ (f+g)'(x) & \doteq & f'(\underline{x}) & + & g'(\underline{x}) \end{array}$$

2) & 2 BIS

$$\begin{array}{ccccc} d(fg)(x)(1) & \overset{\text{THM}}{=} & f(\underline{x})\,dg(\underline{x})(1) & + & df(\underline{x})(1) \cdot g(\underline{x}) \\ \| & \Downarrow & \| & & \| \\ (fg)'(\underline{x}) & \overset{\text{THM}}{=} & f(\underline{x}) \cdot g'(\underline{x}) & + & f'(\underline{x}) \cdot g(\underline{x}) \end{array}$$

PROP

CONSEGUENZA DI PROP.

RMK

SIA $n \in \mathbb{Z}^+$, CONSIDERIAMO PER $i = 1, 2, \dots, n$

$$dx_i : \mathbb{R}^n \to \mathbb{R} \quad \text{OVE}$$

LINEARE!!!

$$dx_i(x_1, x_2, \dots, x_n) = x_i$$

EX $n=3$ $\quad dx(x,y,z) = x, \; dy(x,y,z) = y, \; dz(x,y,z) = z.$

NE SEGUE : SE $p : \mathbb{R}^n \to \mathbb{R}$ È POLINOMIALE

$\Rightarrow$ p È DIFFERENZIABILE IN OGNI PUNTO DI $\mathbb{R}^n$!!!

BREAK DOMANDE?

INIZIO ORE 15.05