Wednesday, October 5, 2022 12:20 PM

INIZIO ORE 14.10

THM I FUNZIONALI "COORDINATA"

$\{dx_1, dx_2, \dots, dx_n\}$ SONO BASE "CANONICA"

DELLO SPAZIO DUALE $(\mathbb{R}^n)^*$.

PROOF (SIST. DI GENERATORI)

SIA $\varphi \in (\mathbb{R}^n)^*$ E SIA $v = (v_1, v_2, \dots, v_n) \in \mathbb{R}^n$ $(*)$

$$\overset{!}{=} \sum_{i=1}^{n} v_i \cdot \underline{e}_i \,.$$

DA $(*)$ SEGUE :

VERO $\forall v \in \mathbb{R}^n$!!

$$\varphi(v) = \varphi\left(\sum_{i=1}^{n} v_i \cdot \underline{e}_i\right) \overset{LIN}{=}$$

$$= \sum_{i=1}^{n} \underset{\downarrow}{v_i} \cdot \underset{\underset{\mathbb{R}}{\Uparrow}}{\varphi(\underline{e}_i)} =$$

$$= \sum_{i=1}^{n} dx_i(v) \cdot \varphi(\underline{e}_i) = \sum_{i=1}^{n} \varphi(\underline{e}_i) \cdot dx_i(v)$$

$$\Downarrow$$

$$\varphi = \sum_{i=1}^{n} \underset{\underset{\mathbb{R}}{\Uparrow}}{\varphi(\underline{e}_i)}\, dx_i$$

QED

(LIN. INDIPENDENZA) THESIS

$$(\dagger) \sum_{i=1}^{n} c_i \cdot dx_i = \underset{\varphi}{\underline{\underline{O}}} \in (\mathbb{R}^n)^* \overset{?}{\Longrightarrow} c_1 = c_2 = \dots = c_n = 0 \,.$$

1) VALUTO LA $(\dagger)$ SU $\underline{e}_1$:

$$\sum_{i=1}^{n} c_i \cdot dx_i(\underline{e}_1) = c_1 \cdot \underset{\underset{1}{\shortparallel}}{dx_1(\underline{e}_1)} = \underline{O}(\underline{e}_1) = 0 \in \mathbb{R} \Longrightarrow c_1 = 0$$

2) VALUTO LA $(\dagger)$ SU $\underline{e}_2$:

$$\sum_{i=1}^{n} e_i \cdot dx_i(\underline{e}_2) = e_2 \cdot \underbrace{dx_2(\underline{e}_2)}_{=1} = e_2 = \underline{0}(\underline{e}_2) = 0 \Rightarrow e_2 = 0$$

RIPETO FINO AD n)

n) VALUTO LA $(\dagger)$ SU $\underline{e}_n$:

$$\sum_{i=1}^{n} e_i \cdot dx_i(\underline{e}_n) = e_n \cdot \underbrace{dx_n(\underline{e}_n)}_{=1} = e_n = \underline{0}(\underline{e}_2) = 0 \Rightarrow e_n = 0$$

PERCIÒ $e_1 = e_2 = \dots = e_n = 0$ <u>QED</u>.

PERCIÒ $\forall \varphi \in (\mathbb{R}^n)^*$ SI HA

$$(**) \quad \varphi \overset{!}{=} \sum_{i=1}^{n} \underbrace{\varphi(\underline{e}_i)}_{\in \mathbb{R}} dx_i \qquad ???$$

<u>APPLICAZIONE FOND AL CALCOLO DIFFERENZIALE</u>

$f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$

f DIFFERENZIABILE IN $\underline{x} \in A$.

ALLORA $(**)$ DIVENTA:

$$(\mathbb{R}^n)^* \ni \underbrace{df(\underline{x})}_{\text{DIFF DI } f \text{ IN } \underline{x}} = \sum_{i=1}^{n} \underbrace{df(\underline{x})(\underline{e}_i)}_{\text{TEOR. FOND}} \cdot dx_i$$

$$= \sum_{i=1}^{n} \frac{\partial f}{\partial \underline{e}_i}(\underline{x}) \cdot dx_i \quad \leftarrow \text{DERIVATE PARZIALI}$$

$$\overset{!}{=} \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\underline{x}) \cdot dx_i .$$

<u>IMPLICAZIONI DI $(**)$</u> COME FUNZIONI NELLE VARIABILI $x_1, x_2, \dots, x_n$, CHI SONO LE FUNZ. LINEARI $\varphi: \mathbb{R}^n \to \mathbb{R}$ LINEARE ???

$$\varphi = \varphi(x \quad) \overset{(**)}{=}$$

$$\varphi = f(x_1, x_2, \dots, x_n) =$$

$$\overset{!}{=} \sum_{i=1}^{n} \underset{\substack{\uparrow \\ \mathbb{R}}}{\varphi(\underline{e}_i)}\, dx_i (x_1, x_2, \dots, x_n)$$

$$\Vert$$

$$= \sum_{i=1}^{n} \underset{\substack{\uparrow \\ \mathbb{R}}}{\varphi(\underline{e}_i)}\, x_i \quad \Leftarrow \text{POLINOMI OMOGENEI DI GRADO 1 (SENZA TERMINE NOTO).}$$

IN $n = 3$

$$\varphi(x, y, z) = -3x + 17y - 27z \qquad \text{LINEARE}$$

$$f(x, y, z) = -3x + 17y - 27z + 1 \qquad \text{NON LINEARE}$$

$$g(x, y, z) = x^2 y + y z^3 \qquad \text{NON LINEARE}$$

QUESTO GENERALIZZA QUANTO GIÀ SAPEVAMO PER $n = 1$!

$\varphi : \mathbb{R} \to \mathbb{R}$

$$\varphi \text{ LINEARE} \implies \varphi(x) = k \cdot x$$

NEL CASO DIFFERENZIALE DIVENTA

$$df(\underline{x}) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\underline{x}) \cdot x_i$$

COME POLINOMIO NELLE VARIABILI $x_1, x_2, \dots, x_n$

ES $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x, y, z) = x^2 y + y^2 z^3$ POLINOMIALE $\Downarrow$ DIFFERENZIABILE

$$\underline{x} = (1, 1, 1)$$

CHI È $df(\underline{x})$?

$$\frac{\partial f}{\partial x}(x, y, z) = 2xy \Big/_{(1,1,1)} = 2$$

$$\frac{\partial f}{\partial y}(x, y, z) = x^2 + 2yz^3 \Big/_{(1,1,1)} = 3$$

$$\frac{\partial f}{\partial z}(\dots$$

$$\frac{\partial f}{\partial z}(x,y,z) = 3y^2z^2\Big/_{(1,1,1)} = 3$$

$$df(\underline{x}) \stackrel{!}{=} 2\,dx + 3\,dy + 3\,dz \quad \text{in } (\mathbb{R}^3)^*$$

$$= 2x + 3y + 3z$$

<u>BREAK</u> DOMANDE?

<u>INIZIO ORE 15.10</u>