Wednesday, October 5, 2022 12:22 PM

PREMESSA: FUNZIONI DI UNA VARIABILE A VALORI VETTORIALI IN $\mathbb{R}^n$.

CIOÈ, $r : A \subseteq \mathbb{R} \to \mathbb{R}^n$,

$t \in A \subseteq \mathbb{R}$ $\qquad r(t) = (r_1(t), r_2(t), \dots, r_n(t)) \in \mathbb{R}^n$

COMP. SCALARI DELLA FUNZ. A VALORI VETTORIALI $r : A \subseteq \mathbb{R} \to \mathbb{R}^n$

OVE $\forall i = 1, 2, \dots, n$, $r_i : A \subseteq \mathbb{R} \to \mathbb{R}$ VALORI SCALARI.

QUINDI $\quad r \equiv (r_1, r_2, \dots, r_n)$

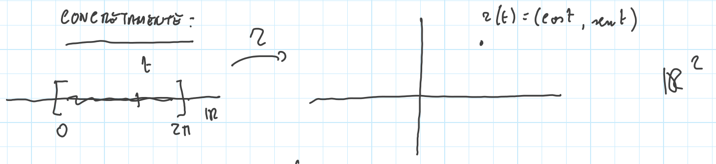
COMP. SCALARI

EX $n = 2$ $\qquad$ SIA $\quad r : [0, 2\pi] \subseteq \mathbb{R} \to \mathbb{R}^2$ t.c.

$r(t) = (\cos t, \operatorname{sen} t) \in \mathbb{R}^2$ $\quad$ CIOÈ

$r_1(t) = \cos t$, $\quad r_2(t) = \operatorname{sen} t$.

CONCRETAMENTE:

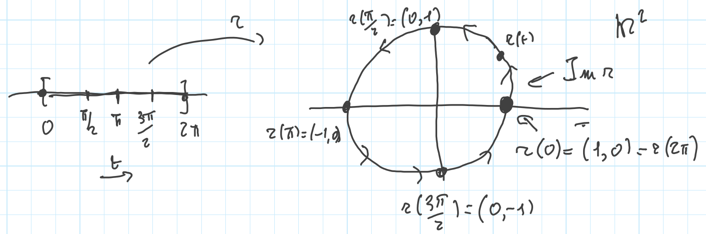


CHI SARÀ $\operatorname{Im} r = \{ r(t);\ t \in [0, 2\pi] \}$?

RICORDIAMO $\qquad (\cos t)^2 + (\operatorname{sen} t)^2 = 1 \qquad$ CIOÈ SODDISFANO

$$x^2 + y^2 = 1$$

$\operatorname{Im} r$ = CIRC. UNITARIA CENTRATA NELL'ORIGINE



SIANO

1) $r : A \subseteq \mathbb{R} \to \mathbb{R}^n$ A APERTO, $a \in A$

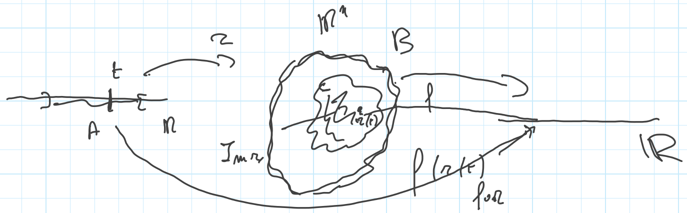
<u>HP 1</u> LE COMP. SCALARI $r_1, r_2, \dots, r_n$ SIANO DERIVABILI IN $a \in A$, CIOE'

$\exists\, r_1'(a), r_2'(a), \dots, r_n'(a)$.

2) $f : B \subseteq \mathbb{R}^n$, B APERTO, $b \in B$

<u>HP 2</u> f DIFFERENZIABILE IN $b \in B$.

SIA $\quad Im\, r = \{ r(t) ; t \in A \} \subseteq B$



PERCIO' RESTA DEFINITA:

$$(f \circ r)(t) \overset{DEF}{=} f(r(t)) \qquad \forall t \in A \text{ !!!}$$

SIA $b = r(a)$

<u>ALLORA</u> <u>THESIS</u>

1) $f \circ r : A \subseteq \mathbb{R} \to \mathbb{R}$ <u>FUNZIONE DI UNA VARIABILE</u> A VALORI SCALARI

È DERIVABILE IN $a \in A \subseteq \mathbb{R}$

<u>DI PIU'</u>

2) $$\exists\, (f \circ r)'(a) \overset{THM}{=} \langle \underbrace{\text{grad} f(r(a))}_{\text{grad} f(b)}, (r_1'(a), r_2'(a), \dots, r_n'(a)) \rangle$$

<u>CASO PARTICOLARE PER $n = 1$</u>

$f : B \subseteq \mathbb{R} \to \mathbb{R}$ DIFF IN $b \in B$

$\Downarrow$

f AMMETTE DERIVATA IN b $\left(\exists\ f'(b)\right)$

DIMO'

$$(f \circ r)'(a) = \langle \operatorname{grad} f(b),\ (r_1'(a), \dots, r_n'(a)) \rangle$$

$\underline{n=1}$ $\quad \operatorname{grad} f(b) = \operatorname{grad} f(r(a)) = \left(f'(r(a))\right) \in \mathbb{R}$

$\xrightarrow{\quad}$ $(r_1'(a), \dots, r_n'(a)) = (r'(a))$

$\underline{n=1}$

$$(f \circ r)'(a) = \left\langle \left(f'(r(a))\right),\ (r'(a)) \right\rangle = f'(r(a)) \cdot r'(a)$$