Thursday, October 13, 2022 11:40 AM

PROBLEMA DELLE DERIVATE MISTE (DEL SECONDO ORDINE)

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$. PER $i, j = 1, 2, \dots, n$

HP1 $\exists \dfrac{\partial f}{\partial x_i}$ SU UN INTORNO $I(\underline{x}, \delta) \subseteq \mathbb{R}^n$

HP2 $\exists \dfrac{\partial}{\partial x_j}\left(\dfrac{\partial f}{\partial x_i}\right) = \dfrac{\partial^{(2)} f}{\partial x_j \partial x_i}$

HP1 BIS $\exists \dfrac{\partial f}{\partial x_j}$ SU UN INTORNO $I(\underline{x}, \delta') \subseteq \mathbb{R}^n$

HP2 BIS $\exists \dfrac{\partial}{\partial x_i}\left(\dfrac{\partial f}{\partial x_j}\right) = \dfrac{\partial^{(2)} f}{\partial x_i \partial x_j}$

PROBLEMA: IN GENERALE, VI È QUALCHE RELAZIONE TRA $\dfrac{\partial^{(2)} f}{\partial x_j \partial x_i}$ E $\dfrac{\partial^{(2)} f}{\partial x_i \partial x_j}$???

IN GENERALE: NO !!!

CONTROESEMPIO $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^2 y + |y|$

$\exists \dfrac{\partial f}{\partial x}(x,y) = 2xy \implies$

$\implies \exists \dfrac{\partial^{(2)} f}{\partial y \partial x}(x,y) \overset{DEF}{=} \dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}(x,y)\right) =$

$= \dfrac{\partial}{\partial y}(2xy) = 2x \quad \forall (x,y) \in \mathbb{R}^2$

MA $\cancel{\exists} \dfrac{\partial f}{\partial y}(x,y) = \dfrac{\partial}{\partial y}(x^2 y + |y|)$ PER $y = 0$!!!

SE $(x,y) = (x,0)$ $\nexists \dfrac{\partial^{(2)} f}{\partial x \partial y}(x,0)$!!!

$y = 0$

BREAK (STOP) DOMANDE?

Break (Stop) Dominate: