Wednesday, October 5, 2022 12:21 PM

INIZIO ORE 11.15

$f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$

DATI $i, j = 1, 2, \dots$, $i \neq j$

$$\frac{\partial^{(2)} f}{\partial x_j \partial x_i}(\underline{x}) \quad , \quad \frac{\partial^{(2)} f}{\partial x_i \partial x_j}(\underline{x}) \qquad ???$$

THM (SCHWARTZ, "DI SCAMBIABILITÀ")

$f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$.

HP1 $\exists \dfrac{\partial^{(2)} f}{\partial x_j \partial x_i}$, $\exists \dfrac{\partial^{(2)} f}{\partial x_i \partial x_j}$ IN ALMENO UN INTORNO $I(\underline{x}, \delta)$

HP2 RIGUARDATE COME FUNZIONI SULL'INTORNO, SIA $\dfrac{\partial^{(2)} f}{\partial x_j \partial x_i}$ CHE $\dfrac{\partial^{(2)} f}{\partial x_i \partial x_j}$ SIANO CONTINUE NEL PUNTO $\underline{x} \in A$.

TH ALLORA

$$\frac{\partial^{(2)} f}{\partial x_j \partial x_i}(\underline{x}) = \frac{\partial^{(2)} f}{\partial x_i \partial x_j}(\underline{x}) \qquad !!!$$

EX $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^3 y + x y^2$

$$\frac{\partial f}{\partial x}(x,y) = 3x^2 y + y^2 \implies$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}(x,y)\right) = 3x^2 + 2y = \frac{\partial^{(2)} f}{\partial y \partial x}(x,y)$$

UGUALI

$$\frac{\partial f}{\partial y}(x,y) = x^3 + 2xy \implies$$

$$\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}(x,y)\right) = 3x^2 + 2y = \frac{\partial^{(2)} f}{\partial y \partial x}(x,y)$$

MAX/MIN LOCALI

$f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, $A \subseteq \mathbb{R}^n$, $\underline{x} \in A$

$\underline{x}$ MAX (MIN) LOCALE PER f $\overset{DEF}{\Longleftrightarrow}$

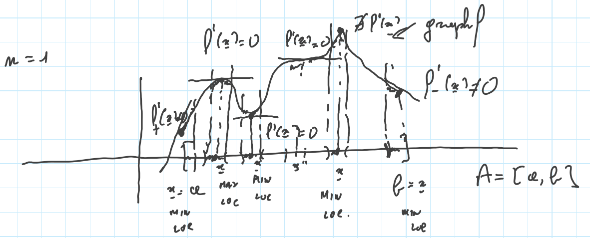
$\exists\ I(\underline{x},\delta) \subseteq \mathbb{R}^n$ t.c.

$f(\underline{x}) \geq f(x) \quad \forall x \in I(\underline{x},\delta) \cap A$ MAX

$(f(\underline{x}) \leq f(x) \quad \forall x \in I(\underline{x},\delta) \cap A)$ MIN

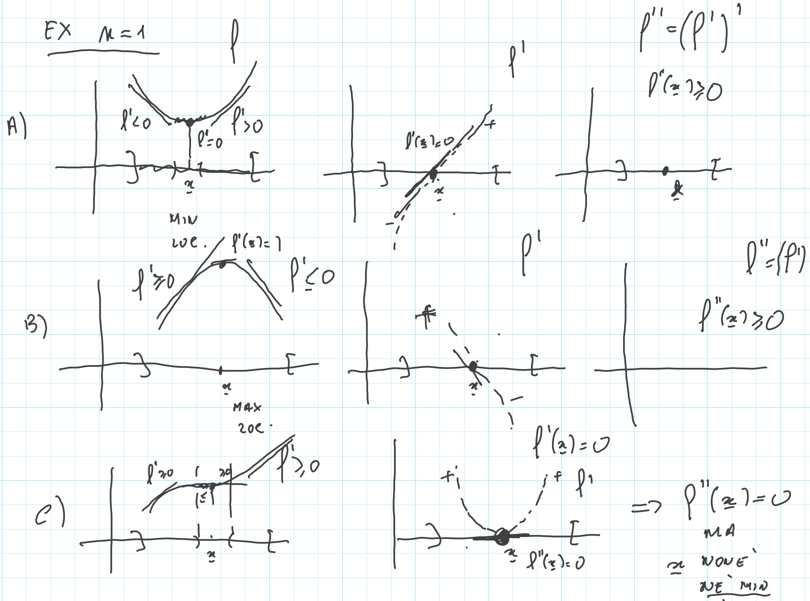
È CHIARO CHE SE A APERTO $\Longrightarrow$ CONDIZIONE $I(\underline{x},\delta)\cap A \longrightarrow I(\underline{x},\delta)$

EX $n=1$



$A=[a,b]$

EX $n=1$



$\Longrightarrow f''(\underline{x})=0$

MA

$\underline{x}$ NON È NÉ MIN NÉ MAX

$\underline{x}$ È DI FLESSO.