Wednesday, October 5, 2022 12:23 PM

MAIN CONSTRUCTION

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$.

SIA $v : \|v\| = 1$ UNA DIREZIONE FISSATA "ARBITRARIA"

E DATO $\underline{x} \in A$ SIA

$r_{\underline{x},v} :]-\delta, \delta[\subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$ t.c.

δ "SUFF. PICCOLO"

$t \in]-\delta,\delta[$ $\quad r_{\underline{x},v} : t \longmapsto \underline{x} + t v \in \mathbb{R}^n$

ORA

$$r_{\underline{x},v}(t) = \underline{x} + tv = (\underline{x}_1 + tv_1, \underline{x}_2 + tv_2, \dots, \underline{x}_n + tv_n) \in \mathbb{R}^n$$

$r_{\underline{x},v,1}(t)$ $\quad r_{\underline{x},v,2}(t)$ $\quad r_{\underline{x},v,n}(t)$

$$r_{\underline{x},v} \equiv (r_{\underline{x},v,1}, r_{\underline{x},v,2}, \dots, r_{\underline{x},v,n})$$

COMP. SCALARI

$i = 1,2,\dots,n$ $\qquad r_{\underline{x},v,i}(t) = \underline{x}_i + t v_i$

$\Rightarrow \exists$ $\quad r'_{\underline{x},v,i}(t) = v_i$ $\quad$ E DI PIU'

$$(r'_{\underline{x},v,1}(t), \dots, r'_{\underline{x},v,n}(t)) = (v_1, v_2, \dots, v_n) = v$$

LA DIREZIONE ORIGINALE !!!

BREAK DOMANDE?

INIZIO 12.15

PER SEMPLICITA', SIA f DIFFERENZIABILE SU TUTTO $A \subseteq \mathbb{R}^n$.

SIA $\operatorname{Im} r'_{z,v} \subseteq A$ $\quad$ A DOMINIO DI f

PERCIÒ

$$F_{z,v} \overset{DEF}{=} f \circ r_{z,v} : \,]-\delta,\delta[\subseteq \mathbb{R} \longrightarrow \mathbb{R}$$

OVE $F_{z,v}(t) = (f \circ r_{z,v})(t) \overset{DEF}{=} f(r_{z,v}(t)) = f(z + tv)$

ORA $\underset{(1)}{f \text{ DIFF}}$ $+$ $\underset{(2)}{\exists\, (r'_{z,v,1}(t), \dots, r'_{z,v,n}(t))} = (v_1, \dots, v_n) = v$

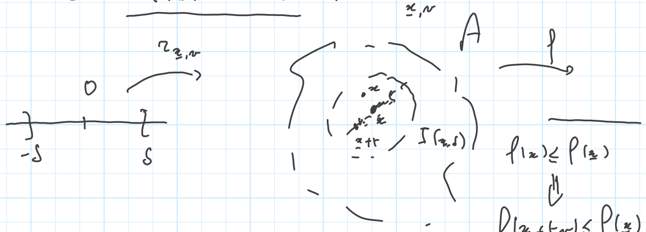
<u>TEOR COMP</u> $\Longrightarrow \exists\, F'_{z,v}(t) \overset{DEF}{=} (f \circ r_{z,v})'(t)$

$$\overset{TMM}{=} \langle \operatorname{grad} f(r_{z,v}(t)), (r'_{z,v,1}(t), \dots, r'_{z,v,n}(t)) \rangle$$

$$= \langle \operatorname{grad} f(z + tv), v \rangle \; !!!$$

MA ORA z <u>MAX (MIN) LOCALE</u> PER $f \Longrightarrow$

$\Longrightarrow 0$ <u>MAX (MIN) LOCALE</u> PER $F'_{z,v}$



$f(x) \leq f(z)$

$\Downarrow$

$f(z+tv) \leq f(z)$

PERCIÒ

$F_{z,v} : \,]-\delta,\delta[\longrightarrow \mathbb{R}$ FUNZ DI UNA VARIABILE

z MAX (MIN) LOCALE PER $f \Longrightarrow$

$0 \in \,]-\delta,\delta[$ MAX (MIN) LOCALE PER $F_{z,v} \Longrightarrow$

$\Longrightarrow F'_{z,v}(0) = 0 \; !!!$ MAX (MIN)

$$\overset{TMM}{=} \langle \operatorname{grad} f(z), v \rangle \; e$$

$$\langle \operatorname{grad} f(z), \underset{\text{VERSORE}}{v} \rangle = 0 \; !!!$$

v ARBITRARIO $\Longrightarrow$

$\underline{x}$ MAX (MIN) LOCALE PER f $\Longrightarrow$

$\langle \text{grad} f(\underline{x}), v \rangle = 0$ $\forall v$ VERSORE

CIOÈ $\|v\| = 1$

$$\text{grad} f(\underline{x}) = \left(\frac{\partial f}{\partial x_1}(\underline{x}), \dots, \frac{\partial f}{\partial x_n}(\underline{x}) \right) = (0, 0, \dots, 0) = \underline{0} \in \mathbb{R}^n$$

$$\Downarrow$$

$df(\underline{x}) = \underline{\underline{0}} \in (\mathbb{R}^n)^*$ FUNZIONALE IDENTICAMENTE NULLO!!!

SE $\underline{x}$ t.c. $df(\underline{x})$ NULLO SI DICE CHE

$\underline{x}$ È PTO. CRITICO. !!!

THM $\underline{x}$ MAX (MIN) LOCALE PER $f \Longrightarrow \underline{x}$ È CRITICO.