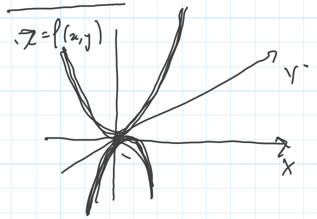
Tuesday, October 18, 2022 11:52 AM

EX3 $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^2 - y^2$

GRAFICAMENTE $z = x^2$



$f(x,0) = x^2$

$f(0,y) = -y^2$

(x,y) CRITICO $\iff$ grad $f(x,y) = (0,0)$

$$(2x, -2y)$$

$$\Downarrow$$

$$x = y = 0$$

$$\Downarrow$$

$\exists!$ $\underline{x}$ CRITICO, $\underline{x} = (0,0)$.

$$H_f(\underline{x}) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \quad \text{DIAGONALE}$$

$$\Downarrow$$

GLI AUTOVALORI SONO

$$\lambda_1 = 2 > 0 \quad \& \quad \lambda_2 = -2 < 0$$

$\Rightarrow H_f(\underline{x})$ <u>NON</u> SEMIDEFINITA $\Rightarrow$

$\underline{x} = (0,0)$ <u>È DI SELLA !!!</u>.

FUNZIONI DIFFERENZIABILI

$f: A \subseteq \mathbb{R}^2 \to \mathbb{R}^n$, A APERTO DI $\mathbb{R}^n$.

CIOÈ

$$f \equiv (f_1, f_2, \dots, f_m)$$

COMP. SCALARI

DEF $\underline{x} \in A \subseteq \mathbb{R}^2$

f DIFF IN $\underline{x} \in A \overset{DEF}{\iff}$

$\exists\, L_{\underline{x}} : \mathbb{R}^2 \longrightarrow \mathbb{R}^m$ LINEARE (OPERATORE)

t.c.

$$\lim_{h \to \underline{0} \in \mathbb{R}^2} \frac{f(\underline{x}+h) - f(\underline{x}) - L_{\underline{x}}(h)}{\|h\|} = \underline{0} \in \mathbb{R}^m \quad (+)$$

SI NOTI CHE $L_{\underline{x}} : \mathbb{R}^2 \longrightarrow \mathbb{R}^m$ OPERATORE LINEARE A VALORI IN $\mathbb{R}^m$

CIOÈ

$$L_{\underline{x}} \equiv (L_{\underline{x},1}, L_{\underline{x},2}, \dots, L_{\underline{x},m}) \quad \text{OVE}$$

$$L_{\underline{x},i} : \mathbb{R}^2 \longrightarrow \mathbb{R} \quad \text{FUNZ. LINEARE!}$$

(+) È EQUIVALENTE A:

$$(++) \begin{cases} \lim_{h \to \underline{0} \in \mathbb{R}^2} \dfrac{f_1(\underline{x}+h) - f_1(\underline{x}) - L_{\underline{x},1}(h)}{\|h\|} = 0 \in \mathbb{R} \\ \quad - \quad - \quad - \quad - \\ \lim_{h \to \underline{0} \in \mathbb{R}^2} \dfrac{f_m(\underline{x}+h) - f_m(\underline{x}) - L_{\underline{x},m}(h)}{\|h\|} = 0 \in \mathbb{R} \end{cases}$$

IN BREVE

f ... $(f : A \subseteq \mathbb{R}^2 \to \mathbb{R}^m)$ ... $\underline{x} \iff$

f DIFF ($f: A \subseteq \mathbb{R}^2 \to \mathbb{R}^n$) IN $\underline{x}$ ⟺

f_i DIFF. ($f_i: A \subseteq \mathbb{R}^2 \to \mathbb{R}$) IN $\underline{x}$ E DI PIU'

$$df_i(\underline{x}) = L_{\underline{x},i} \qquad \forall i = 1,2,\dots,n$$

$L_{\underline{x}}: \mathbb{R}^2 \to \mathbb{R}^n$ <u>OPERATORE LINEARE</u>

FISSATE BASI <u>CANONICHE SIA IN $\mathbb{R}^2$ CHE $\mathbb{R}^n$</u>

$L_{\underline{x}}$ SARA' DESCRITTO DA UNA ED UNA SOLA MATRICE

$$M_{L_{\underline{x}}}$$

DI ORDINE $n \times 2$!!!

ORA, PER DEFINIZIONE, SI AVRA'

$$\begin{array}{c} M_{L_{\underline{x}}} \\ \begin{pmatrix} & & \\ & ? & \\ & & \end{pmatrix} \end{array} \times \begin{array}{c} h \\ \begin{pmatrix} h_1 \\ \vdots \\ h_n \end{pmatrix} \end{array} = \begin{pmatrix} L_{\underline{x},1}(h) \\ \vdots \\ L_{\underline{x},i}(h) \\ \vdots \\ L_{\underline{x},n}(h) \end{pmatrix} \leftarrow (L_{\underline{x}}(h))^t$$

ORA, DATO $i = 1,2,\dots,n$ CHI SARA'

$$L_{\underline{x},i}(h) = \langle \operatorname{grad} f_i(\underline{x}), h \rangle \quad !!!!$$

$$= \sum_{j=1}^{2} \frac{\partial f_i}{\partial x_j}(\underline{x}) \cdot h_j$$

PERCIO'

$$\text{1 RIGA?} \quad \begin{array}{c} M_{L_{\underline{x}}} \\ \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(\underline{x}) & \dots & \frac{\partial f_1}{\partial x_2} \\ & & \end{pmatrix} \end{array} \times \begin{pmatrix} h_1 \\ \vdots \\ \vdots \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{2} \frac{\partial f_1}{\partial x_j}(\underline{x}) \cdot h_j \\ \vdots \end{pmatrix}$$

$$\text{ultima}\begin{pmatrix} \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1}(\underline{x}) & \cdots & \frac{\partial f_m}{\partial x_n}(\underline{x}) \end{pmatrix} \begin{pmatrix} \vdots \\ h_n \end{pmatrix} \qquad \begin{pmatrix} \vdots \\ \sum_{j=1}^{n} \frac{\partial f_m}{\partial x_j}(\underline{x}) \cdot h_j \end{pmatrix}$$

$\downarrow$

MATRICE JACOBIANA !!!

$$J_{f(\underline{x})} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(\underline{x}) & \cdots & \frac{\partial f_1}{\partial x_n}(\underline{x}) \\ \text{---} & & \text{-} \\ \frac{\partial f_m}{\partial x_1}(\underline{x}) & \cdots & \frac{\partial f_m}{\partial x_n}(\underline{x}) \end{pmatrix} = \begin{pmatrix} \operatorname{grad} f_1(\underline{x}) \\ \vdots \\ \operatorname{grad} f_m(\underline{x}) \end{pmatrix}.$$