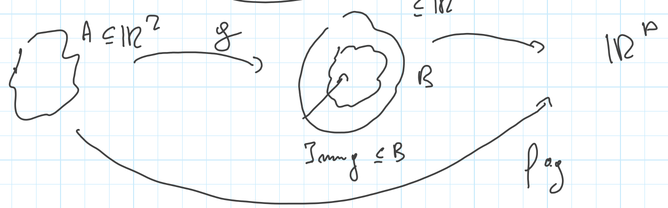
THM (TEOREMA DI COMPOSIZIONE - CASO GENERALE)

SIA $g : A \subseteq \mathbb{R}^2 \to \mathbb{R}^n$, A APERTO

SIA $f : B \subseteq \mathbb{R}^n \to \mathbb{R}^p$, B APERTO

SIA $\underline{\text{Imm}\, g = \{ g(x) \in \mathbb{R}^n ; x \in A \} \subseteq B}$
$\subseteq \mathbb{R}^n$



OVE $(f \circ g)(x) = f(g(x))$ $\qquad x \in A \subseteq \mathbb{R}^2$

SI HA $f \circ g \subseteq A \subseteq \mathbb{R}^2 \to \mathbb{R}^p$.

SIA $a \in A$, $b = g(a) \in B$.

<u>HP 1</u> g DIFF IN $a \in A$

<u>HP 2</u> f DIFF IN $b = g(a)$.

<u>TH 1</u> $f \circ g : A \subseteq \mathbb{R}^2 \to \mathbb{R}^p$ È DIFFERENZIABILE IN $a \in A$

<u>TH 2</u> $d(f \circ g) : \mathbb{R}^2 \to \mathbb{R}^p$ LINEARE È TALE CHE

$$d(f \circ g)(a) = df(b) \circ dg(a) \qquad (*)$$

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "dg(a)", "\text{LIN}"'] \arrow[rr, bend right] & \mathbb{R}^n \arrow[r, "df(b)", "\text{LIN}"'] & \mathbb{R}^p \end{tikzcd}$$

$df(b) \circ dg(a) \overset{\text{THM}}{=} d(f \circ g)(a) : \mathbb{R}^2 \to \mathbb{R}^p$ <u>LINEARE</u>

OVVIAMENTE $(*)$, IN TERMINI MATRICIALI:

PRODOTTO RIGA / COLONNA

$$J_{(f \circ g)(a)} = J_{f(b)} \times J_{g(a)}$$

$\uparrow$ HA ORDINE $\qquad\qquad \downarrow$

$$p \times r = (p \times m) \times (m \times r)$$

CASO PART. $\quad r = 1 \quad , p = 1$

$g : A \subseteq \mathbb{R} \to \mathbb{R}^n \quad , g = (g_1, \dots, g_m) \; , g_i : A \subseteq \mathbb{R} \to \mathbb{R}^n$

DIFF $\Rightarrow \exists \; g_i'(a) \quad \forall i = 1, 2, \dots, m$ DERIVABILI

$f : B \subseteq \mathbb{R}^n \to \mathbb{R}$ $\qquad$ DIFF IN $b = g(a)$

$\overset{\text{THM}}{\Longrightarrow}$ $f \circ g$ DIFF IN $a \in A \subseteq \mathbb{R}$

$f \circ g : A \subseteq \mathbb{R} \to \mathbb{R} \quad \Downarrow \quad f \circ g$ DERIVABILE IN $a \in A \subseteq \mathbb{R}$

E DI PIU'

$J_{(f \circ g)(a)} : \mathbb{R}^1 \to \mathbb{R}^1$ LINEARE

1×1

$$\left((f \circ g)'(a) \right) \overset{\text{THM}}{=} \underset{(1 \times m)}{J_{f(b)}} \times \underset{(m \times 1)}{J_{g(a)}}$$

$$= \left(\frac{\partial f}{\partial x_1}(b), \dots, \frac{\partial f}{\partial x_m}(b) \right) \times \begin{pmatrix} g_1'(a) \\ \vdots \\ g_m'(a) \end{pmatrix} =$$

CIOE' $\displaystyle = \sum_{i=1}^{m} \frac{\partial f}{\partial x_i}(b) \cdot g_i'(a) =$

$$= \langle \operatorname{grad} f(b), (g_1'(a), \dots, g_m'(a)) \rangle$$

TEOREMA DELLE FUNZIONI IMPLICITE

TEOREMA DI DINI

BREAK DOMANDE :

INIZIO ORE 15.10