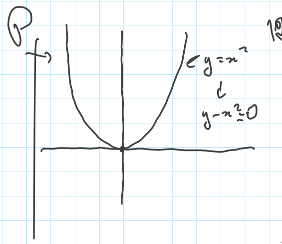
Wednesday, October 5, 2022 12:26 PM

PROBLEM / EXAMPLE IN $\mathbb{R}^2$



$\mathcal{P} = \operatorname{graph} f =$ ove $f:\mathbb{R}\to\mathbb{R}$

$y=f(x)=x^2$

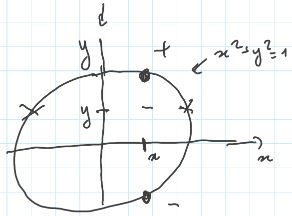
$\stackrel{DEF}{=} \{(x,y)\in\mathbb{R}^2;\ (x,y)=(x,f(x))=(x,x^2)\}$

DESCRIZIONE PARAMETRICA DI $\mathcal{P}$

$\stackrel{DEF}{=} \{(x,y)\in\mathbb{R}^2;\ y-x^2=0\}$

DESCRIZIONE CARTESIANA

SIA $\mathcal{C} = \{(x,y)\in\mathbb{R}^2;\ x^2+y^2-1=0\}$ DESCR. CARTESIANA

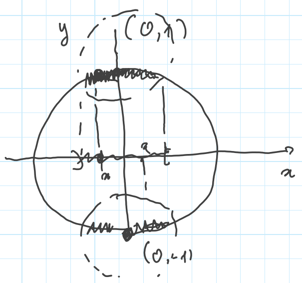


$\mathcal{C}$ AMMETTE DESCRIZIONE PARAMETRICA ?

NOTIAMO

$y = \pm\sqrt{1-x^2}$ ← NON SENSE!!!

$I((0,1),\delta)$



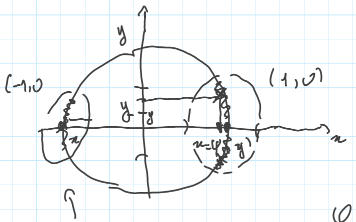
LOCALIZZANDO, CIOÈ

$\mathcal{C} \cap I((0,1),\delta)$

SARANNO I PTI

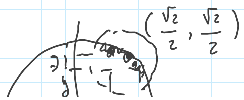
$y = f(x) = +\sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

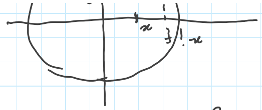


$x = \psi(y) = +\sqrt{1-y^2}$

$x = \varphi(y) = -\sqrt{1-y^2}$



$y = +\sqrt{1-x^2}$



$x = \pm\sqrt{1-y^2}$

SIA $F: \mathbb{R}^2 \to \mathbb{R}$, $F(x,y) = x^2+y^2-1$, $F \in C^{(1)}_{\mathbb{R}^2}$

E QUINDI $\mathcal{C} = \{(x,y) \in \mathbb{R}^2;\ F(x,y)=0\}$ CARTESIANA

ORA

$\to \frac{\partial F}{\partial x}((0,1)) = 2x\big|_{x=0} = 0$, $\frac{\partial F}{\partial y}((0,1)) = 2y\big|_{y=1} = 2 \neq 0$!

NB $y = f(x)$

$\to \frac{\partial F}{\partial x}((0,-1)) = 0$, $\frac{\partial F}{\partial y}((0,-1)) = -2 \neq 0$!

$\frac{\partial F}{\partial x}((1,0)) = 2x\big|_{x=1} = 2 \neq 0$!, $\frac{\partial F}{\partial y}((1,0)) = 2y\big|_{y=0} = 0$

$x = \varphi(y)$!!!

$\frac{\partial F}{\partial x}((-1,0)) = -2 \neq 0$!, $\frac{\partial F}{\partial y}((-1,0)) = 0$

$\frac{\partial F}{\partial x}\left(\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)\right) \neq 0$, $\frac{\partial F}{\partial y}\left(\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)\right) \neq 0$

$\Downarrow$ $\Downarrow$

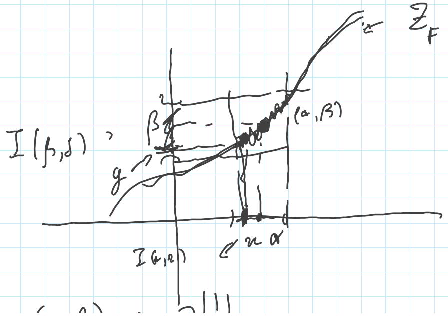
$x = f(y)$ $\qquad y = \varphi(x)$

TEOREMA DI DINI $F: \mathbb{R}^2 \to \mathbb{R}$, $F \in C^{(1)}$

SIA $Z_F = \{(x,y) \in \mathbb{R}^2;\ F(x,y)=0\}$ DESCR. CARTESIANA

SIA $(\alpha,\beta) \in Z_F$ CIOÈ $F(\alpha,\beta) = 0$.

$Z_F \subseteq \mathbb{R}^2$



SIA $\frac{\partial F}{\partial}$ ...

$\dfrac{\partial F}{\partial y}(\alpha,\beta) \neq 0$ ...!

ALLORA $\exists\, I(\beta,\delta)$, $\exists\, I(\alpha,r)$

t.c. $\forall x \in I(\alpha,r)$ $\exists!\, y \in I(\beta,\delta)$ t.c.

$(x,y) \in Z_F$!!!

$Z_F \cap (I(\alpha,r) \times I(\beta,\delta))$

CIOÈ POSSO (LOCALMENTE) SCRIVERE

$y = f(x)$!!!

DI PIÙ $f \in C^{(1)}_{I(\alpha,r)}$

DI PIÙ ANCORA

$$f'(\alpha) \overset{TAM}{=} - \frac{\dfrac{\partial F}{\partial x}((\alpha,\beta))}{\dfrac{\partial F}{\partial y}((\alpha,\beta))}$$

$\leftarrow$ PER HP $\neq 0$

<u>GEOMETRICAMENTE</u> SIA $Z_F = \{(x,y) \in \mathbb{R}^2 ;\ F(x,y) = 0\}$

$F \in C^{(1)}$

E SIA F DI TIPO DINI, CIOÈ

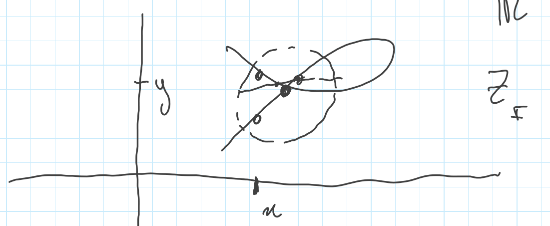
$\forall (\alpha,\beta) \in Z_F$ SI HA ALMENO

$\dfrac{\partial F}{\partial x}(\alpha,\beta) \neq 0$ OVVERO $\dfrac{\partial F}{\partial y}(\alpha,\beta) \neq 0$

COME SARÀ FATTO $Z_F \subseteq \mathbb{R}^2$???

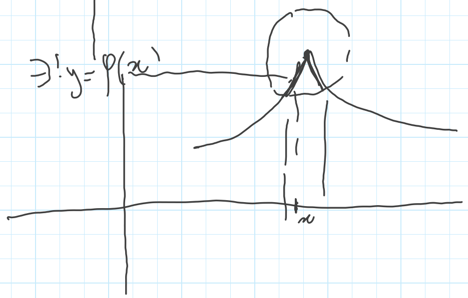
PUÒ ESSERE COSÌ ?

1)



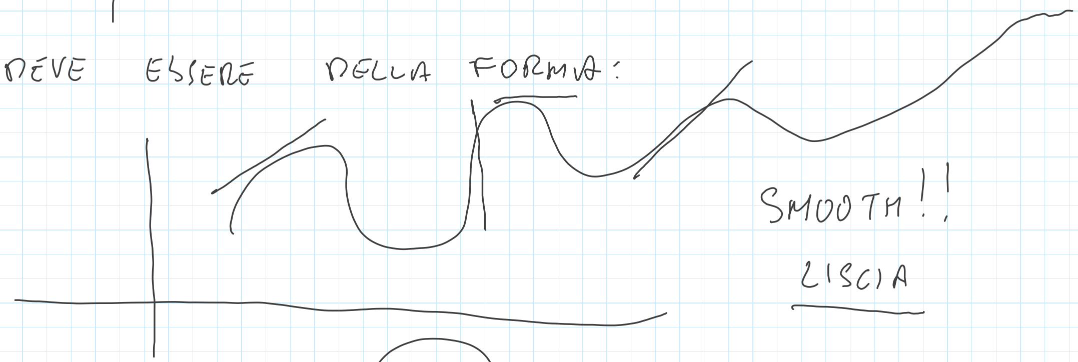
NO!!!

2)



NO!!!

Z_F DEVE ESSERE DELLA FORMA:



SMOOTH!!

LISCIA