DINI'S THEOREM

$F: A \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$ $\quad$ A APERTO, $\quad F \in C^{(1)}_A$

SIA $\quad Z_F \overset{\text{def}}{=} \{(x,y)\in\mathbb{R}^2 \,;\, F(x,y)=0\}$

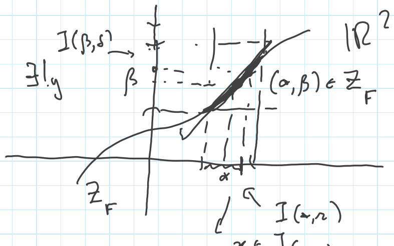
SIA $\quad (\alpha,\beta)\in Z_F \quad$ (CIOÈ $F(\alpha,\beta)=0$).

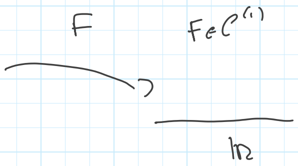
SE $\quad \dfrac{\partial F}{\partial y}(\alpha,\beta)\neq 0 \quad \Rightarrow$

$\Rightarrow \quad \exists\, I(\alpha,r),\ \exists\, I(\beta,\delta) \quad$ t.c.

$\forall x \in I(\alpha,r) \quad \exists!\, y\in I(\beta,\delta) \quad$ t.c.

$F(x,y)=0 \quad$ (CIOÈ $(x,y)\in Z_F$)





CIOÈ, SI PUÒ (LOCALMENTE)

$$y = f(x)$$

DI PIÙ $\quad \exists\, f'(\alpha) = - \dfrac{\frac{\partial F}{\partial x}(\alpha,\beta)}{\frac{\partial F}{\partial y}(\alpha,\beta)} \quad \frac{\partial F}{\partial y}(\alpha,\beta) \neq 0$

IMPLICAZIONI GEOMETRICHE

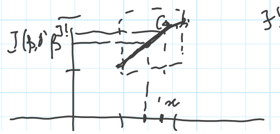
SIA $\quad V = \{(x,y)\in\mathbb{R}^2 \,;\, F(x,y)=0\} \quad$ CON $F\in C^{(1)}$

SIA $\quad (\alpha,\beta)\in V \quad$ CIOÈ $\quad F(\alpha,\beta)=0 \iff (\alpha,\beta)\in Z_F = V$

COM'È FATTO V "VICINO AL PTO (α,β)"?

1) SE $\quad \dfrac{\partial F}{\partial y}(\alpha,\beta)\neq 0$

LOCALMENTE



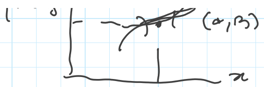
$\exists!\, y$ t.c. $F(x,y)=0$

CIOÈ

$y = f(x)$

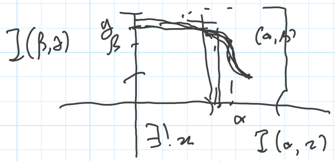
$\exists\, f'(\alpha)$

CIOÈ



con $\exists$ | (α)

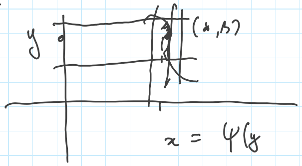
2) $\dfrac{\partial F}{\partial x}(\alpha,\beta) \neq 0$



t.e. $F(x,y)=0$ cioè $x = \varphi(y)$

con $\exists\, \varphi'(\beta)$

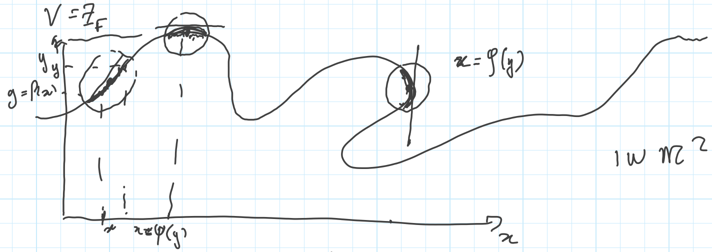
cioè

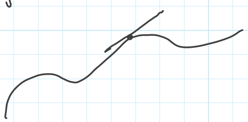


$x = \varphi(y$

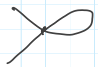
SE $\dfrac{\partial F}{\partial y}(\alpha,\beta) \neq 0$, $\dfrac{\partial F}{\partial x}(\alpha,\beta) \neq 0$ posso fare in entrambi i modi

cioè se F è di "tipo Dini" sarà della forma:

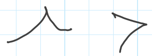




cioè proibito 1)



2)



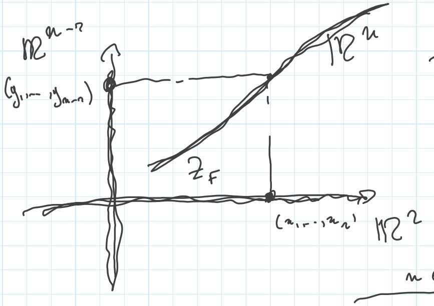
TEOR. DELLE FUNZ IMPLICITE (DINI GENERALE)

I VERSIONE

THM 1 $F: A \subset \mathbb{R}^n \to \mathbb{R}^{n-k}$ A aperto $F \in C^{(1)}$

SIA Z_F IL LUOGO DEGLI ZERI DI F.

$$\mathbb{R}^n = \mathbb{R}^2 \times \mathbb{R}^{n-2}$$



$\xrightarrow{F} \mathbb{R}^{n-2}$

$Z_F = \{(x_1, \dots, x_2; y_1, \dots, y_{n-2}) \in \mathbb{R}^n;$
$F(x_1, \dots, x_2, y_1, \dots, y_{n-2}) = \underline{0}\}$

UN PTO IN $\mathbb{R}^n = ((x_1, \dots, x_2), \underbrace{(y_1, \dots, y_{n-2})}_{}) $ n COORDINATE

$$Z_F = \{(x_1, \dots, x_2, y_1, \dots, y_{n-2}) \in \mathbb{R}^n;\ F(x_1, \dots, x_2, y_1, \dots, y_{n-2}) = \underline{0} \in \mathbb{R}^{n-2}\}$$

SIA $(\alpha, \beta) = (\alpha_1, \dots, \alpha_2, \beta_1, \dots, \beta_{n-2}) \in \mathbb{R}^n$ t.c $F(\alpha, \beta) = \underline{0}$
CIOÈ
$(\alpha, \beta) \in Z_F$

SIA

$$J_{F(\alpha,\beta)} = \begin{pmatrix} \frac{\partial F_1}{\partial x_1}(\alpha,\beta) & \dots & \frac{\partial F_1}{\partial x_2}(\alpha,\beta) & \frac{\partial F_1}{\partial y_1}(\alpha,\beta) & & \frac{\partial F_1}{\partial y_{n-2}}(\alpha,\beta) \\ \hline & & & & & \\ \frac{\partial F_{n-2}}{\partial x_1}(\alpha,\beta) & \dots & \frac{\partial F_{n-2}}{\partial x_2}(\alpha,\beta) & \frac{\partial F_{n-2}}{\partial y_1}(\alpha,\beta) & \dots & \frac{\partial F_{n-2}}{\partial y_{n-2}}(\alpha,\beta) \end{pmatrix}$$

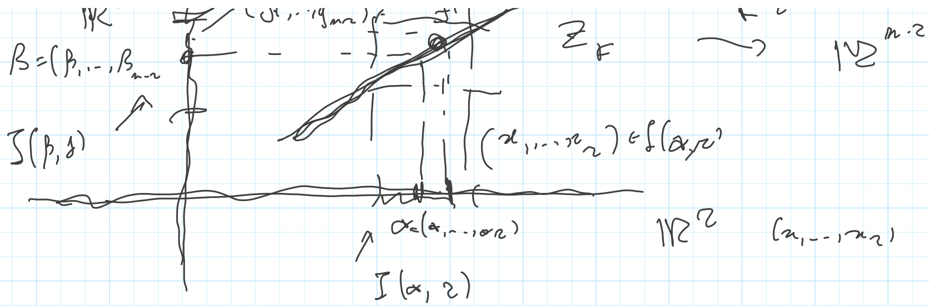
QUADRATO DI ORDINE
$(n-2) \times (n-2)$

SIA HP! $\det \neq 0$

ALLORA $\exists\, I(\underbrace{\alpha_1, \dots, \alpha_2}_{\alpha}; r)$ $\exists\, I(\underbrace{\beta_1, \dots, \beta_{n-2}}_{\beta}, \delta)$ t.c

$$\forall (x_1, \dots, x_2) \in I(\alpha_1, \dots, \alpha_2; r) \quad \exists!\, (y_1, \dots, y_{n-2}) \in I(\beta_1, \dots, \beta_{n-2}; \delta)$$

t.c $F(x_1, \dots, x_2; y_1, \dots, y_{n-2}) = 0$ CIOÈ



CIOÈ $(y_1, \dots, y_{n-r}) = \varphi(x_1, \dots, x_r)$ SCALARMENTE.

$$\begin{cases} y_1 = \varphi_1(x_1, \dots, x_r) \\ \dots \\ y_{n-r} = \varphi_{n-r}(x_1, \dots, x_r) \end{cases} \qquad \to \varphi : I(\alpha, r) \to \mathbb{R}^{n-r}$$

E DI PIÙ $\varphi_1, \varphi_2, \dots, \varphi_{n-r}$ SONO ANCORA DI CLASSE $C^{(1)}$

HP È RIMPIAZZATA DA:

$$rk\left(J_{F(\alpha,\beta)}\right) = n - r$$

$n-r$ COLONNE TALI CHE

$$J_{F(\alpha,\beta)} = \begin{array}{c} \begin{array}{ccccccc} x_1 & \dots & y_1 & y_2 & y_i & y_{n-r} & x_2 \end{array} \\ \left(\begin{array}{ccccccc} & & & & & & \end{array} \right) \end{array}$$

$\Leftarrow$ ABBIA DETERMINANTE $\neq 0$

$$y_1 = f_1(x_1, \dots, x_r)$$

$$y_{n-r} = f_n(x_1, \dots, x_r)$$

$$f_1, \dots, f_{n-r} \in C^{(1)}$$

INIZIO 12.20