Wednesday, October 5, 2022 12:29 PM

$V \subseteq \mathbb{R}^n$ VARIETÀ REGOLARE DI DIMENSIONE r E CLASSE $C^{(1)}$

$\Updownarrow$ DEF

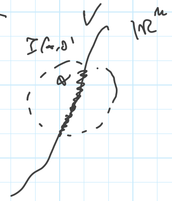
$\forall \alpha \in V \quad \exists I(\alpha,\delta) \subseteq \mathbb{R}^n, \exists F : I(\alpha,\delta) \subseteq \mathbb{R}^n \to \mathbb{R}^{n-r}$

$F \in C^{(1)}_{I(\alpha,\delta)}$ t.c.

1) $V \cap I(\alpha,\delta) = \{ x \in I(\alpha,\delta) ; F(x) = \underline{0} \in \mathbb{R}^{n-r} \}$

SCALARMENTE $\Updownarrow$

SIST. DI $n-r$ EQS IN n INDETERMINATE
$$\begin{cases} F_1(x_1,\dots,x_n) = 0 \\ \dots \\ F_{n-r}(x_1,\dots,x_n) = 0 \end{cases}$$



2) REGOLARITÀ

$rk(J_{F(\alpha)}) = n-r$ (max) !!

TANGENZA

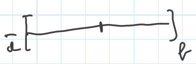
CURVA IN $\mathbb{R}^n$

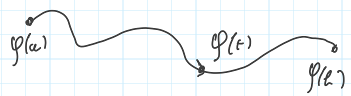
$\varphi : [a,b] \to \mathbb{R}^n$, $\varphi \in C^{(1)}_{[a,b]}$ CURVA DI CLASSE $C^{(1)}$

$\varphi \equiv (\varphi_1, \varphi_2, \dots, \varphi_n)$, $\varphi_i \in C^{(1)}$.

DATA $\underbrace{\varphi}_{t}$ CURVA, SIA $supp(\varphi) = \{ \varphi(t) \in \mathbb{R}^n ; t \in [a,b] \}$

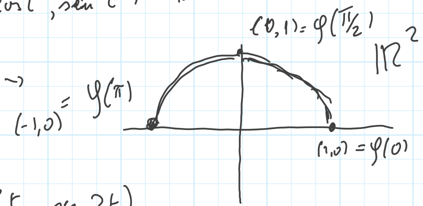
$\varphi \to \varphi(t) = (\varphi_1(t), \dots, \varphi_n(t)) \in \mathbb{R}^n$





EX $\varphi : [0,\pi] \to (\cos t, \sin t) \in \mathbb{R}^2$

$supp(\varphi) \to$

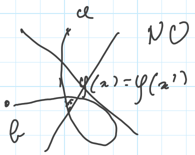


$\psi : [0, \pi/2] = (\cos 2t, \sin 2t)$

$supp(\psi) = supp(\varphi)$

– φ CURVA SEMPLICE E APERTA $\overset{DEF}{\iff}$

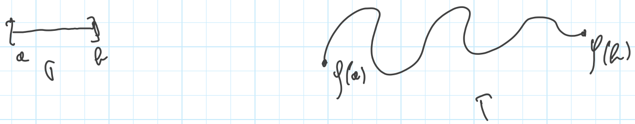
1) $\varphi : [a,b] \subseteq \mathbb{R} \xrightarrow[1-1]{su} supp(\varphi)$



2) φ OMEOMORFISMO TRA $[a,b]$ E $supp(\varphi)$

CIOÈ

φ CONTINUA (VERO PER HP) $+ (\exists)\varphi^{-1} : supp(\varphi) \to [a,b]$ CONTINUA !!!



RMK SE HO $\varphi : (X, d_x) \to (Y, d_y)$ SP. METRICI

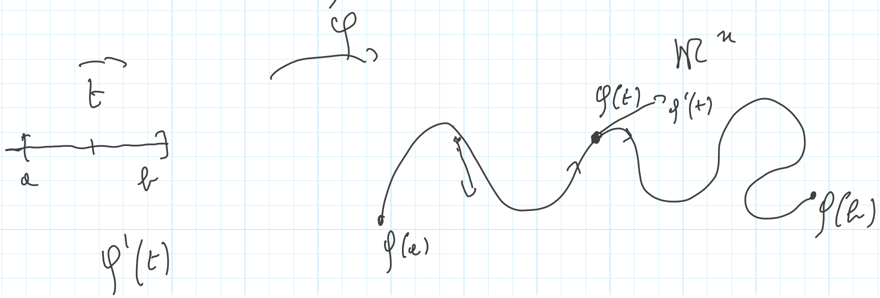
φ INVERTIBILE, φ CONTINUA

SE (X, d_x) COMPATTO $\implies \varphi^{-1}$ CONTINUA

MA $A \subseteq \mathbb{R}^n$ EUCLIDEO A COMPATTO $\iff$ A LIMITATO & CHIUSO.

$\varphi : [a,b] \to \mathbb{R}^n$ CURVA SEMPLICE/APERTA SI DIRÀ REGOLARE $\iff$

$\forall t \in \,]a,b[\,, \; \varphi'(t) = (\varphi_1'(t), \dots, \varphi_n'(t)) \neq \underline{0} \in \mathbb{R}^n$



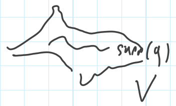
VETTORE $h \in \mathbb{R}^n$ TANGENTE ALLA VARIETÀ $V \subseteq \mathbb{R}^n$ NEL PUNTO $\alpha \in V$.

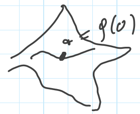
DEF $\quad h \in \mathbb{R}^n$ TG. A V IN $\underline{\alpha} \in V \overset{DEF}{\iff}$

$\exists\, \varphi : [-\delta, \delta] \longrightarrow \mathbb{R}^n$ CURVA SEMPLICE/APERTA + REGOLARE

t.c.

1) $\operatorname{supp} \varphi \subseteq V$ $\qquad$ 2) $\varphi(0) = \alpha$





3) $h = \varphi'(0) = (\varphi_1'(0), \varphi_2'(0), \dots, \varphi_n'(0)) \neq \underline{0}$!!!

"INSIEME" (?) TANGENTE A V IN $\alpha \in V$:

$T(\alpha) \overset{DEF}{=} \{ h \in \mathbb{R}^n ;\ h \text{ TG A } V \text{ in } \underline{\alpha} \} \cup \{\underline{0}\}$ $\quad$ È OPERATIVA