Tuesday, November 8, 2022 11:35 AM

METRIC SPACES

$\rightarrow (X, d)$ WHERE

X: SET

d: METRIC FUNCT (DISTANCE)

$d: X \times X \longrightarrow \mathbb{R}$ S.T. (DISTANCE)

i) $x, x' \in X \quad d(x,x') \geq 0$ &

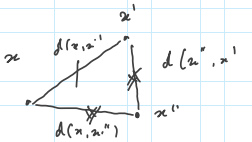
$d(x,x') = 0 \iff x = x'$

ii) $d(x,x') = d(x',x)$ SYMMETRY PROP.

iii) $\forall x, x', x'' \in X$ WE HAVE

$d(x,x') \leq d(x,x'') + d(x'',x')$ (TRIANGULAR INEQUALITY)

THAT IS

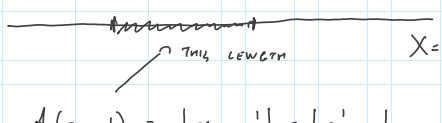


EX $X = \mathbb{R}$ SET $x, x' \in \mathbb{R}$

REAL NUMBERS

$d(x,x') \overset{\text{DEF}}{=} |x - x'|$ METRIC FUNCT

EUCLIDEAN METRIC FUNCT.



$X = \mathbb{R}$

$d(x,x') = |x - x'| = |x' - x|$

EX 2 $n \in \mathbb{Z}^+$, $X = \mathbb{R}^n$

GIVEN $\underline{x} = (x_1, x_2, \ldots, x_n)$, $y = (y_1, \ldots y_n) \in \mathbb{R}^n$

SET $d(\underline{x},\underline{y}) \overset{DEF}{=} \sqrt{(x_1-y_1)^2+(x_2-y_2)^2+\dots+(x_n-y_m)^2}$ (*)

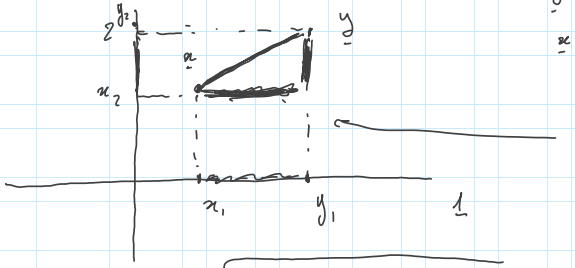
↑ IS METRIC FUNCT (EUCLIDEAN DISTANCE IN DIMENSION n)

TWO SUBCASES OF EX 2

1) $n=1$ $\underline{x}=(x)$, $\underline{y}=(y)$ THEN

$d(\underline{x},\underline{y}) = \sqrt{(x-y)^2} \overset{MB}{=} |x-y|$ EX 1

2) $n=2$ $X=\mathbb{R}^2$ $\underline{y}=(y_1,y_2)$ $\underline{x}=(x_1,x_2)$



$(x_1-y_1)^2$

$(x_2-y_2)^2$

$d(\underline{x},\underline{y}) \overset{DEF}{=} \sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$ (PITAGORA THM) VVV...

EX 3 DISCRETE SPACES

(X, d) WHERE IS

X SET, d METRIC FUNCT

$x,x' \in X$

$$d(x,x') = \begin{cases} 0 & x=x' \\ 1 & x\neq x' \end{cases}$$

METRIC FUNCT

DISCRETE METR. FUNCT

OPEN SPHERICAL NEIGHBOURHOOD (X,d) METRIC SPACE

GIVEN $x\in X$, $r\in\mathbb{R}^+$ (POS REAL NUMBERS)

$$\mathbb{I}(x,r) \overset{DEF}{=} \{x'\in X;\ d(x',x)<r\}$$

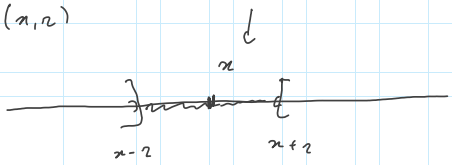
(x CENTER, r RADIUS)

EX $X = \mathbb{R}$

$$I(x,2) = \{x' \in \mathbb{R};\ |x' - x| < 2\}$$

$$= \{x' \in \mathbb{R};\ x-2 < x' < x+2\}$$

$I(x,2)$



EX BIS $\quad X = \mathbb{R}^2$

$\mathbb{R}^2$

$I(x,2) = ?$

$= \{x' \in \mathbb{R}^2;\ d(x',x) < 2\}$

BREAK QUESTIONS?

BEGIN AGAIN AT

10.15