Tuesday, November 8, 2022 11:36 AM

BEGIN AGAIN AT 10.15

CAN YOU HEAR / SEE ?

THE CASE OF DISCRETE SPACES

(X, d) DISCRETE SPACE

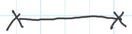
SET DISCRETE METRIC FUNCT, THAT IS

$$d(x, x') \overset{DEF}{=} \begin{cases} 0 & x = x' \\ 1 & x \neq x' \end{cases}$$

WHAT IS $I(x, r) = ???$

FIX $x \in X$ AND $r \in \mathbb{R}^+$

$$I(x, r) = \begin{cases} \{x\} & r \leq 1 \\ X & r > 1 \end{cases}$$



OPEN SET (X, d) METRIC SPACE

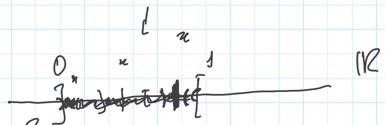
$A \subseteq X$

A OPEN $\overset{DEF}{\iff}$ $\forall$ (FOR EVERY) $x \in A$ $\exists r \in \mathbb{R}^+$ S.T.

$I(x, r) \subseteq A$.

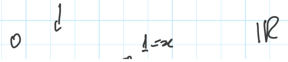
EX $X = \mathbb{R}$ EUCLIDEAN

i) $A =]0, 1[\overset{DEF}{=} \{x \in \mathbb{R} ; 0 < x < 1\}$ GRAPHICAL WAY $=$



IS A OPEN ? YES

ii) $B =]0, 1] = \{x \in \mathbb{R} ; 0 < x \leq 1\}$



IS NOT CONTAINED IN B!!!

IS B OPEN?

SET $x \in B$, $x = 1$

THE COND. (OF OPEN) FAILS FOR THE POINT

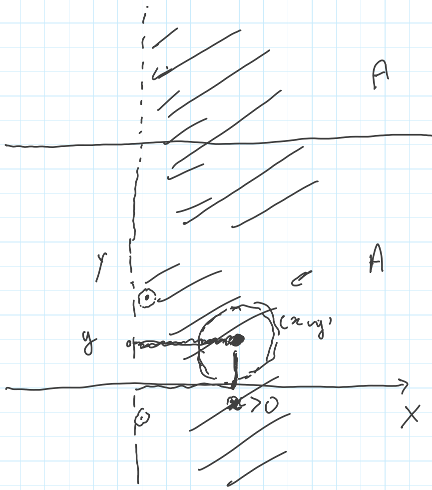
$$x = 1 \in B =]0,1]$$

$\Rightarrow B =]0,1]$ IS NOT OPEN ???

EX IN $X = \mathbb{R}^2$ EUCLIDEAN

i) $A = \{(x,y) \in \mathbb{R}^2 ; x > 0\}$ THAT IS $X = \mathbb{R}^2$

A IS IT OPEN? YES



HOW CAN WE CHOOSE THE RADIUS $r \in \mathbb{R}^+$?

NOTICE THAT $x > 0$!!!

SO FOR EVERY

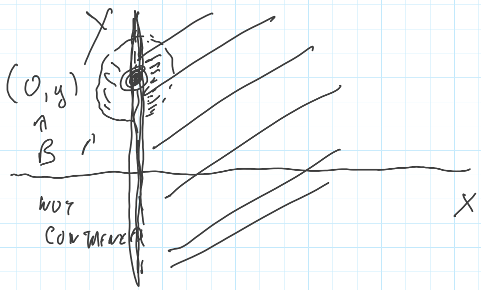
$0 < r \leq x$ EVERYTHING IS OK

THAT IS $x > 0$

$$I((x,y), r)) \subseteq A.$$

ii) $B = \{(x,y) \in \mathbb{R}^2 ; x \geq 0\}$; IS IT OPEN?

$X = \mathbb{R}^2$



B HALF PLANE

THEN, THE COND TO BE OPEN FAILS

FOR ANY POINT OF THE FORM

$$(0, y) \in B.$$

THEREFORE, B NOT OPEN !!!

DISCRETE SPACES ???

(X, d) DISCRETE SPACE.

LET $A \subseteq X$

FIX ANY $x \in A$. NOTICE THAT

$I(x, r) = \{x\}$ WITH $r \leq 1$

BUT, $x \in A \iff \{x\} \subseteq A$ HENCE

$\forall x \in A$ WE HAVE

$I(x, r) = \{x\} \subseteq A$ WITH $r \leq 1$

SO, THE COND TO BE OPEN IS VALID !!!

IN DISCRETE SPACES, EVERY SUBSET $A \subseteq X$ IS OPEN !!!

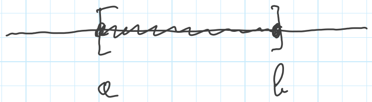
3) CLOSED SETS (X, d) METRIC SPACE

GIVEN $A \subseteq X$

A CLOSED $\overset{\text{DEF}}{\iff} A^c \overset{\text{DEF}}{=} X \setminus A$ IS OPEN

EX IN $\mathbb{R}$ EUCLIDEAN, GIVEN $a, b \in \mathbb{R}$, $a < b$

SET $A = [a, b] \overset{\text{DEF}}{=} \{ x \in \mathbb{R} ; a \le x \le b \}$



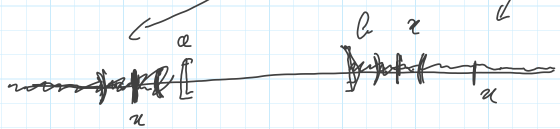
QUESTION IS $A = [a,b]$ CLOSED ??? YES

THIS IS EQUIVANT TO:

IS $A^c = \mathbb{R} \setminus [a,b]$ IS OPEN ??? YES

BUT

$$A^c = \mathbb{R} \setminus [a,b] = \{ x \in \mathbb{R} ; x < a \} \dot{\cup} \{ x \in \mathbb{R} ; x > b \}$$

$A^c =$ 

BUT, GIVEN $x \in A^c$ EITHER $x \in]-\infty, a[$

OR $x \in]b, +\infty[$

THEN $A^c = \mathbb{R} - [a,b]$ IS OPEN $\Rightarrow$ A IS CLOSED !!!

$\parallel$
A

STOP QUESTIONS ???

BYE BYE