Tuesday, November 8, 2022 11:36 AM

HELLO !!!

BEGIN AT 14.10, OK ???

CAN YOU SEE / HEAR ME ???

FEEDBACK PLEASE ..

OK, LET US BEGIN !?

(X, d) METRIC SPACE $\longrightarrow$

$\longrightarrow$ OPEN SPHERICAL NEIGH.., OPEN SETS, CLOSED SETS...

ACCUMULATION POINTS (X, d) M.S.

GIVEN $x_0 \in X$, AND $B \subseteq X$.

DEF. x_0 IS AN ACCUMULATION POINT FOR THE SUBSET $B \subseteq X$ $\overset{DEF}{\Longleftrightarrow}$

(FOR EVERY) $\to$ $\forall\, r \in \mathbb{R}^+$ WE HAVE :

ϕ = EMPTY SET

$$(*) \quad \left(I(x_0, r) \setminus \{x_0\} \right) \cap B \neq \phi$$

CENTER

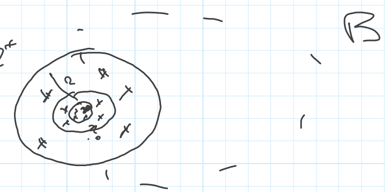
"ARBITRARY" RADIUS

THE INTERSECTION IS "ALWAYS" NOT EMPTY !!!

PRACTICAL EXAMPLE

x_0 ACC. POINT FOR A SUBSET $B \subseteq X$

$\forall\, r \in \mathbb{R}^+$

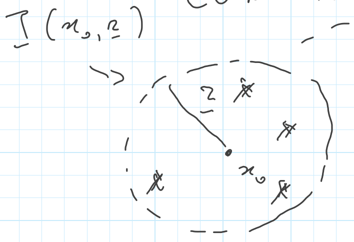


(CROSSED POINT $\times$ ARE POINTS OF B)

RMK LET x_0 BE ACC. POINT FOR $B \subseteq X$.

FIX A RADIUS, THAT $r \in \mathbb{R}^+$. AND

CONSIDER THE FOLLOWING SITUATION:

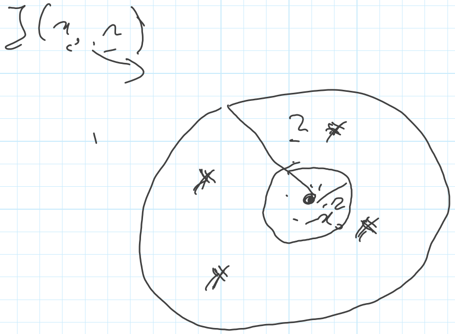


B BY (a)

(CROSSED POINTS ARE POINTS OF B)

IS IT POSSIBLE THAT WE FIND A FINITE NUMBER OF CROSSED POINT? NO!!!

ASSUME THAT THE ASSERTION IS TRUE, THAT IS:



$\underline{r}$ FIXED RADIUS $\underline{r} \in \mathbb{R}^+$

$\times$ CROSSED POINTS IN B!!!

r WITH

$r <$

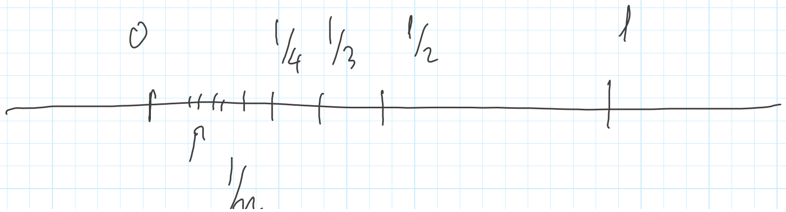
$< \min d(x_0, x)$

THIS CONTRADICTS THE DEF OF ACC. POINT!

EX 1 IN $\mathbb{R}$ EUCLIDEAN

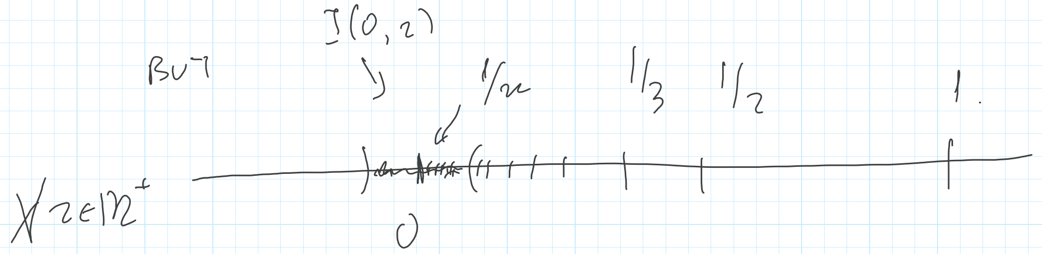
$$A = \left\{ \frac{1}{n} \; ; \; n \in \mathbb{Z}^+ \right\}$$

TO MAKE A PICTURE:



NOW, NOTICE THAT $0 \notin A \overset{\text{DEF}}{=} \{ 1/n \,;\, n \in \mathbb{Z}^+ \}$

BUT



0 IS AN ACCUMULATION POIN FOR

$A = \{ 1/n \,;\, n \in \mathbb{Z}^+ \}$!!!

<u>NOTICE THAT</u> : 0 ACC. POINT FOR A,

BUT $0 \notin A$.

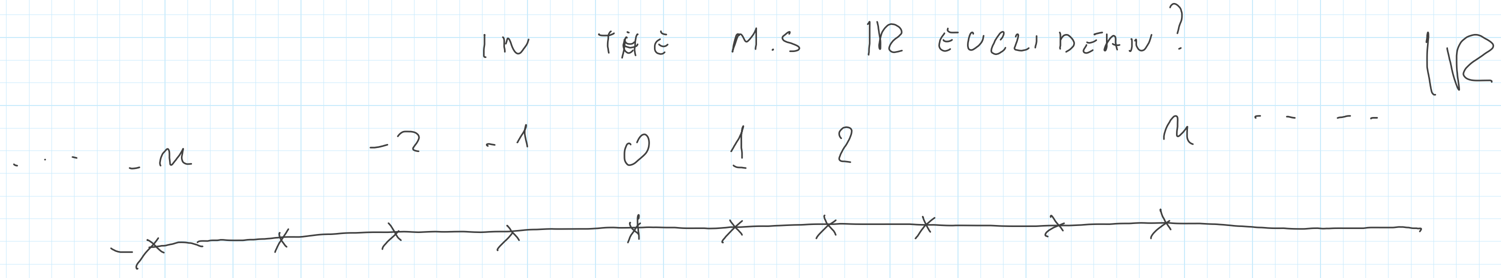
<u>EX 2</u> IN $\mathbb{R}$ EUCLIDEAN.

CONSIDER $A = \mathbb{Z} \subseteq \mathbb{R}$.

INTEGER POINTS

DOES $\mathbb{Z}$ ADMIT ACCUMULATION POINTS

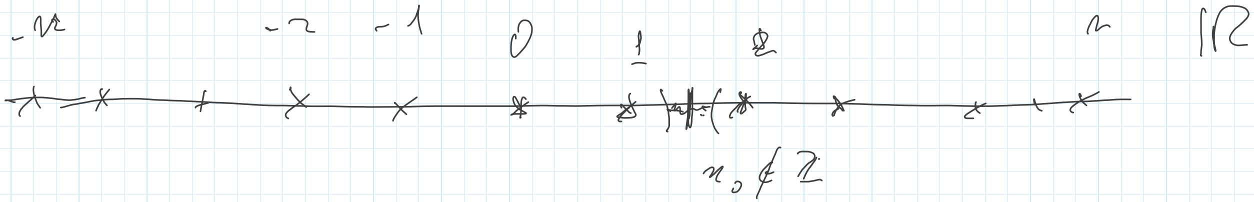
IN THE M.S $\mathbb{R}$ EUCLIDEAN?



FIX A POINT $x_0 \in \mathbb{R}$. WE HAVE

TWO OPTIONS:

i) $x_0 \notin \mathbb{Z}$. GRAPHICALLY, WE HAVE



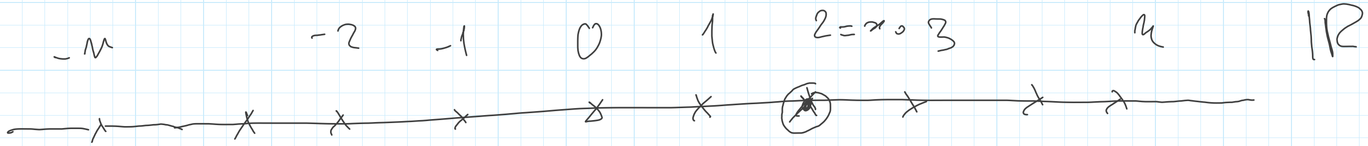
IS IT AN ACC. POINT FOR $\mathbb{Z}$? <u>NO</u>

$r < \min\left(d(x_0, 1), d(x_0, 2)\right)$

WE HAVE: $I(x_0, r) \cap \mathbb{Z} = \emptyset$!!!

THEREFORE, $x_0 \notin \mathbb{Z}$ IS <u>NOT</u> ACC. POINT.

ii) $x_0 \in \mathbb{Z}$. (SAY, $x_0 = 2$)



IS $x_0 = 2$ AN ACC. POINT FOR $\mathbb{Z}$? <u>NO</u> !!!

WHY? IF WE TAKE <u>$r \leq 1$</u>

$I(\quad) \cap \mathbb{Z} \quad \{ x_0 \; 2 \}$

$I(x_0, 2) \cap \mathbb{Z} = \{x_0 = 2\}$

BUT IN THE COND, WE HAVE TO REMOVE THE CENTER !!!

THEN $\qquad 2 \leq 1$

$$\left(\underline{I(x_0 = 2, 2) \setminus \{x_0 = 2\}}\right) \cap \mathbb{Z} = \emptyset \quad ???$$

HENCE, $\mathbb{Z} \subseteq \mathbb{R}$ HAS <u>NO</u> ACC. POINTS !!!

<u>BREAK</u> <u>QUESTIONS???</u>

<u>BEGIN AT 15.10</u>