Tuesday, November 8, 2022 11:37 AM

FILE ONENOTE OF

LESSON 2BIS

BEGIN AT 15.10

OK, CAN YOU SEE/HEAR ME?

(FEEDBACK, PLEASE)

PROP (X, d) METRIC SPACE, $A \subseteq X$.

A CLOSED $\overset{\text{THM}}{\Longleftrightarrow}$ A CONTAINS ALL ITS ACCUMULATION POINTS.

PROOF $\Longrightarrow$) (PROOF BY CONTRADICTION)

SUPPOSE THAT

$\exists x_0 \notin A$, x_0 ACC. FOR A $\Longleftrightarrow$

$\Updownarrow$

$\exists x_0 \in A^c$ s.t. $\forall r \in \mathbb{R}^+$ WE HAVE

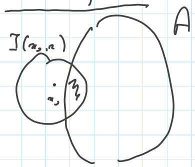
$\forall r \in \mathbb{R}^+$

$$\left(I(x_0, r) \setminus \{x_0\} \right) \cap A \neq \emptyset \quad (*)$$

SINCE $x_0 \in A^c$ $(x_0 \notin A)$ $(*)$ BECOMES EQUIVALENT TO

$(**)$ $\forall r \in \mathbb{R}^+$

$$I(x_0, r) \cap A \neq \emptyset \Leftrightarrow I(x_0, r) \not\subseteq A^c$$



HENCE, WE HAVE:

$(***)$ $\forall r \in \mathbb{R}^+$ WE HAVE

$$I(x_0, r) \not\subseteq A^c \quad \text{(COMPLEMENTARY SET OF A)}$$

BUT $(**)$ IS EQUIVALENT TO

(†) $\nexists\ I(x_0, r) \subseteq A^c$ WITH $x_0 \in A^c$.

$$\Downarrow$$

A^c NOT OPEN

$$\Updownarrow \text{ DEF}$$

<u>A NOT CLOSED</u> (CONTRADICTION !!!)

$\Leftarrow$) (BY CONTRADICTION)

ASSUME THAT <u>A NOT CLOSED</u> $\Leftrightarrow$

A^c <u>IS NOT OPEN</u> $\Rightarrow$

(*) $\exists\, x_0 \in A^c$ FOR WHICH $\nexists\ I(x_0, r) \subseteq A^c \Leftrightarrow$

(**) $\exists\, x_0 \in A^c$ S.T. $\forall r \in \mathbb{R}^+$ WE HAVE

$$I(x_0, r) \not\subseteq A^c$$

$$\Updownarrow$$

(***) $\exists\, x_0 \in A^c$ $\forall r \in \mathbb{R}^+$

$$\boxed{I(x_0, r) \cap A \neq \emptyset}$$

<u>BUT, $x_0 \in A^c$ ($x_0 \notin A$)</u>, THEN

(***) IS EQUIVALENT TO

$\exists\, x_0 \in A^c$ $\forall r \in \mathbb{R}^+$

$$\underline{(I(x_0, r) \setminus \{x_0\}) \cap A \neq \emptyset} \quad \equiv \quad (†)$$

BUT COND (†) MEANS THAT

<u>x_0 IS AN ACC. POINT FOR A</u> ! BUT, WE ASSUMED THAT

$x_0 \in A^c \Leftrightarrow x_0 \notin A$

CONTRADICTION !!! [illegible]

CONTRADICTION ... QED OK, DONE.

GO BACK TO EX $A = \{ 1/n \,;\, n \in \mathbb{Z}^+ \} \subseteq \mathbb{R}$ EUCLIDEAN

QUESTION : IS A CLOSED IN $\mathbb{R}$??

WE KNOW THAT :

1) $0 \notin A$, 2) 0 ACC. POINT FOR A

$\Downarrow$

A NOT CLOSED

FOUR MAIN PROPERTIES OF OPEN & CLOSED SUBSETS W.R.T. UNION / INCLUSION

PROP (X, d) M. SPACE . WE HAVE .

O_1) ANY UNION OF OPEN SETS IS OPEN

O_2) ANY (FINITE !!!) INTERSECTION OF OPEN SETS IS OPEN

C_1) ANY INTERSECTION OF CLOSED SETS IS CLOSED

IS CLOSED

C_2) ANY (FINITE !!!) UNION OF CLOSED SETS

IS CLOSED.

WHY ARE THEY PAIRWISE EQUIVALENT ???

FROM THE HIGH SCHOOL, WE RECALL:

DE MORGAN LAWS $\quad X$ SET,

$\{A_i \subseteq X ; i \in I\}$

THEN:

1) $$\left(\bigcup_i A_i\right)^c \overset{\text{THM}}{=} \bigcap_i A_i^c$$

2) $$\left(\bigcap_i A_i\right)^c = \bigcup_i A_i^c$$

BYE, GOOD

WEEK END