Tuesday, November 8, 2022 11:38 AM

BEGIN AT 14.10

HELLO??

COULD YOU SEE/HEAR ME?

FOUR MAIN PROPS

PROP

i) ANY UNION OF OPEN SETS IS AN OPEN SET (O1)

ii) ANY FINITE INTERSECTION OF OPEN SETS IS AN OPEN SET (O2)

BY DEMORGAN LAWS

iii) ANY FINITE ! UNION OF CLOSED SETS IS A CLOSED SET. (C1)

iv) ANY INTERSECTION OF CLOSED SETS IS A CLOSED SET (C2)

COUNTER EXAMPLES

O2) "WE FORGET" THE FINITNESS HYPS.

IN $\mathbb{R}$ EUCLIDEAN

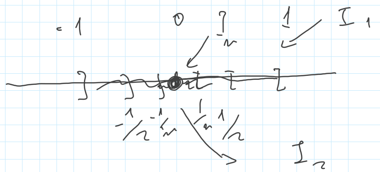
OPEN

FOR $n \in \mathbb{Z}^+$, LET $I_n = \left]-\frac{1}{n}, \frac{1}{n}\right[$.

CONSIDER THE INFINITE INTERSECTION

$$\bigcap_{n \in \mathbb{Z}^+} I_n = \bigcap_{n=1}^{\infty} \underbrace{\left]-\frac{1}{n}, \frac{1}{n}\right[}_{\text{OPEN}} \overset{?}{=} \{0\}$$

SINGLETON SET WHOSE UNIQUE POINT IS $0 \in \mathbb{R}$.



NOT OPEN IN $\mathbb{R}$ EUCLIDEAN

C1) "WE FORGET" THE FINITNESS COND

IN <u>UNION</u> <u>ASSERTION ABOUT CLOSED SETS</u> !!!

CONSIDER, IN $\mathbb{R}$ EUCLIDEAN, THE SETS:

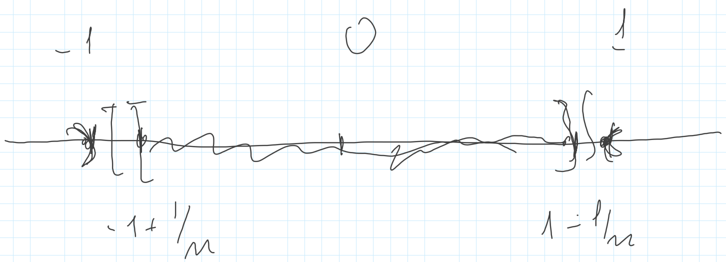
$$J_n = [-1+1/n,\ 1-1/n] \quad , \forall n \in \mathbb{Z}^+ .$$

WHAT IS

$$\bigcup_{n \in \mathbb{Z}^+} J_n = \bigcup_{n=1}^{\infty} [-1+1/n,\ 1-1/n] \stackrel{?}{=}]-1, 1[$$

<u>CLOSED</u> ???

$n \in \mathbb{Z}^+$



<u>THIS IS NOT</u>

<u>CLOSED</u> !!!

<u>SIMPLE (IMPORTANT) DEF</u>

<u>METRIC SUBSPACE</u> (X, d) METRIC SPACE,

AND CONSIDER $A \subset X$ $(A \neq \emptyset)$.

A INHERITS A METRIC SPACE STRUCTURE

FROM (X, d):

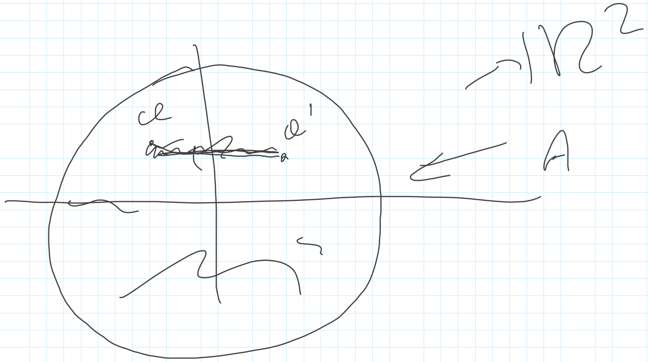
FOR EVERY $a, a' \in A$, WE DEFINE

$$d_{/A}(a, a') \overset{\text{DEF}}{=} d_X(a, a') .$$

CLEARLY, $d_{/A} : A \times A \longrightarrow \mathbb{R}$ IS A METRIC FUNCT

HENCE, $(A, d_{/A})$ IS A METRIC SPACE !

EX $\mathbb{R}^2$ EUCLIDEAN, AND $A = \{(x,y) \in \mathbb{R}^2 ;\ x^2+y^2 \leq 1\}$

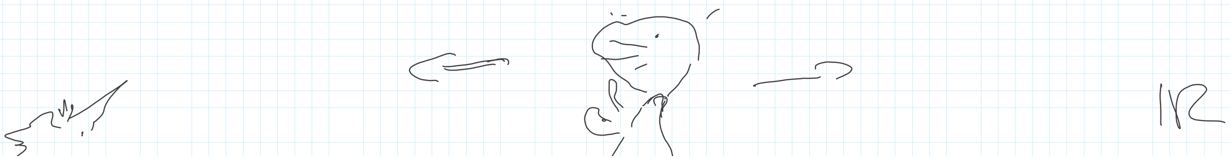


$$d_{/A}(a, a') = d_X(a, a')$$

$X = \mathbb{R}^2$

CONNECTION PROPERTY (INTRODUCTION)

CARTOON (LA "LINEA" / THE "LINE") .



$X = \mathbb{R} \setminus \{0\}$

SUBSPACE

PASSAGE FOR $\mathbb{R}$ EUCLIDEAN $\longrightarrow$ $X = \mathbb{R} \setminus \{0\}$???

WE HAVE THAT

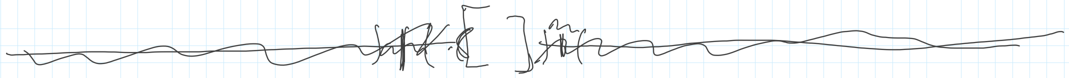
DISJOINT

$$X = \mathbb{R} \setminus \{0\} = \,]-\infty, 0[\ \dot{\cup}\ ]0, +\infty[$$

OPEN (NOT TRIVIAL)

GRAPHICALLY :

OPEN! O O OPEN!



$]-\infty, 0[$ $]0, +\infty[$

HENCE, WE CAN SPLIT

$X = \mathbb{R} \setminus \{0\}$ IN THE DISJOINT UNION

NON TRIVIAL

$$X = \mathbb{R} \setminus \{0\} = \left]-\infty, 0\right[\,\dot{\cup}\, \left]0, +\infty\right[\quad !!!$$

OPEN   OPEN

MAIN DEFINITION (X, d) METRIC SPACE.

(X, d) IS SAID TO "DISCONNECTED" $\overset{\text{DEF}}{\Longleftrightarrow}$

$\exists A_1, A_2 \subseteq X$, $A_1, A_2 \neq \emptyset, X$ (NOT TRIVIAL)

A_1, A_2 OPEN S.T.

$$\begin{cases} (i) & A_1 \cap A_2 = \emptyset \\ (ii) & A_1 \dot{\cup} A_2 = X \end{cases}$$

?!!

TRIVIALLY,

(X, d) IS CONNECTED $\overset{\text{DEF}}{\Longleftrightarrow}$ (X, d) NOT DISCONNECTED.

BREAK   QUESTIONS?