

Corso di Analisi Matematica T-1
Corso di Laurea in Ingegneria dell'Automazione
Anno Accademico 2022/23

Esercizi

A) Scrivere in forma algebrica i seguenti numeri complessi:

1. $\frac{3-i}{4-i}$ 2. $\frac{2-i}{2+i}$ 3. $\frac{4-3i}{(2+i)^2}$ 4. $\frac{(2\sqrt{3}+i)^3}{\sqrt{3}-i}$

B) Scrivere in forma trigonometrica i seguenti numeri complessi:

1. $\sqrt{3}-i$ 6. $(-\sqrt{3}+i)^7$ 10. $\frac{1-i}{(\sqrt{3}-i)^4}$
2. $\frac{1}{-1+i\sqrt{3}}$ 7. $(1+4i)^5$ 11. $\frac{(1+i)^5}{(1-i\sqrt{3})^3}$
3. $-1-3i$ 8. $(-1-2i)^6$ 12. $\frac{(1+2i)(1+i)^8}{(4i)^2}$
4. $-1+3i$
5. $\frac{-1+2i}{4i}$

C) Determinare le radici quadrate e cubiche dei seguenti numeri complessi:

1. -3 2. $-i$ 3. $1-i\sqrt{3}$ 4. $1-i$ 5. $-1+2i$ 6. $2+i$

D) Risolvere le seguenti equazioni in campo complesso:

1. $z^2 + z + 8 = 0$ 14. $\left(\frac{z-i}{2z+i}\right)^2 = 8i$
2. $z^2 + iz - 2 = 0$
3. $z^2 + 2z + 1 + 2i = 0$ 15. $\left(\frac{z^2 + 3iz}{z^2 + 2}\right)^2 = 1$
4. $(3+3i)z^2 + \sqrt{5}(2-2i)z + 1+i = 0$
5. $2z^2 + 2(\sqrt{3}+3i)z - 1+i\sqrt{3} = 0$ 16. $\left(z + \frac{1}{z}\right)^2 = (1-i)^4$
6. $z^2 - i2\sqrt{6}z - i = 0$ 17. $\left(z^2 + \frac{\sqrt{3}}{2} - i\frac{1}{2}\right)^3 = -i$
7. $iz^2 - 4z + 2 - 4i = 0$
8. $z^3 + iz = 0$
9. $z^4 - 4z^2 + 4 + 2i = 0$ 18. $\left(z^2 + 2iz - \frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^3 = 1$
10. $z^6 - 7z^3 - 8 = 0$
11. $(z-i)^6 = -8$ 19. $\left((3+3i)z + \frac{1+i}{z}\right)^2 = 5(1+i)^6$
12. $(z+4)^6 = (z-4)^6$
13. $(z^2 + i2\sqrt{2}z - 1)^2 = -1$ 20. $\left(2iz - \frac{1+2i}{z}\right)^2 = (6-2i)^2$

E) Scrivere in forma algebrica i seguenti numeri complessi:

1. e^{-2+3i} 2. $\exp((2+i)^3)$ 3. $\frac{e^{2+i}}{e^{3-2i}}$ 4. $\exp((1-i)^6)$

F) Scrivere in forma trigonometrica i seguenti numeri complessi:

1. $3e^{2-4i}$ 2. $(e^{3-2i})^2$ 3. $e^{(3-2i)^2}$ 4. $(1-i)e^{2+i}$

G) Risolvere le seguenti equazioni in campo complesso:

1. $e^z = -4i$ 5. $e^{2z} + 6e^z + 9 + 2i = 0$ 9. $(e^{-z} + 1)^3 = -1$
2. $e^z = -3 + 2i$ 6. $e^{iz} + 4e^{-iz} = -2$ 10. $e^{iz} + (1-i)e^{-iz} - i + 2 = 0$
3. $e^{iz} = 2 - 2i$ 7. $e^z + e^{-z} = ie^{-z} + i - 2$ 11. $e^{(1+i)z} = 1 + i$
4. $e^{(2+i)z} = 1$ 8. $(e^{2z} + 4)^2 = (ie^{2z} - 4)^2$ 12. $e^{4iz} + (1-i)e^{2iz} - i = 0$

Soluzioni

A)

1. $\frac{13}{17} - \frac{1}{17}i$

3. $-i$

2. $\frac{3}{5} - \frac{4}{5}i$

4. $\frac{19}{4} + \frac{53\sqrt{3}}{4}i$

B)

1. $2\left(\cos\left(\frac{11}{6}\pi\right) + i\operatorname{sen}\left(\frac{11}{6}\pi\right)\right)$

2. $\frac{1}{2}\left(\cos\left(-\frac{2}{3}\pi\right) + i\operatorname{sen}\left(-\frac{2}{3}\pi\right)\right)$

3. $\sqrt{10}\left(\cos(\arctan 3 + \pi) + i\operatorname{sen}(\arctan 3 + \pi)\right)$

4. $\sqrt{10}\left(\cos(\pi - \arctan 3) + i\operatorname{sen}(\pi - \arctan 3)\right)$

5. $\frac{\sqrt{5}}{4}\left(\cos\left(\arctan\frac{1}{2}\right) + i\operatorname{sen}\left(\arctan\frac{1}{2}\right)\right)$

6. $128\left(\cos\left(-\frac{\pi}{6}\right) + i\operatorname{sen}\left(-\frac{\pi}{6}\right)\right)$

7. $17^{5/2}\left(\cos(5\arctan 4) + i\operatorname{sen}(5\arctan 4)\right)$

8. $5^3\left(\cos(6\arctan 2 + \pi) + i\operatorname{sen}(6\arctan 2 + \pi)\right)$

9. $64(\cos 0 + i\sin 0)$

10. $\frac{1}{8\sqrt{2}}\left(\cos\left(\frac{5}{12}\pi\right) + i\operatorname{sen}\left(\frac{5}{12}\pi\right)\right)$

11. $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4} + i\operatorname{sen}\frac{\pi}{4}\right)$

12. $\sqrt{5}\left(\cos(\arctan 2 + \pi) + i\operatorname{sen}(\arctan 2 + \pi)\right)$

C)

1. $\pm i\sqrt{3}; \frac{\sqrt[3]{3}}{2} + i\frac{3^{5/6}}{2}, -\sqrt[3]{3}, \frac{\sqrt[3]{3}}{2} - i\frac{3^{5/6}}{2}$

2. $\pm\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right); \frac{\sqrt{3}}{2} - i\frac{1}{2}, i, -\frac{\sqrt{3}}{2} - i\frac{1}{2}$

3. $\pm\left(\sqrt{\frac{3}{2}} - i\frac{1}{\sqrt{2}}\right); \sqrt[3]{2}\exp\left(i\frac{5+6k}{9}\pi\right), k=0,1,2$

4. $\pm 2^{1/4} \left(\cos\left(\frac{7}{8}\pi\right) + i \operatorname{sen}\left(\frac{7}{8}\pi\right) \right)$;
 $2^{1/6} \left(\cos\left(\frac{7}{12}\pi\right) + i \operatorname{sen}\left(\frac{7}{12}\pi\right) \right)$, $-\frac{1}{\sqrt[3]{2}} - i \frac{1}{\sqrt[3]{2}}$, $2^{1/6} \left(\cos\left(\frac{23}{12}\pi\right) + i \operatorname{sen}\left(\frac{23}{12}\pi\right) \right)$

5. $\pm 5^{1/4} \exp\left(i \frac{1}{2}(\pi - \arctan 2)\right)$; $5^{1/6} \exp\left(i \frac{1}{3}(\pi - \arctan 2)\right)$, $k = 1, 3, 5$

6. $\pm 5^{1/4} \exp\left(i \frac{1}{2} \arctan \frac{1}{2}\right)$; $5^{1/6} \exp\left(i \frac{1}{3} \arctan \frac{1}{2}\right)$. $k = 0, 1, 2$

D)

1. $-\frac{1}{2} - i \frac{\sqrt{31}}{2}$, $-\frac{1}{2} + i \frac{\sqrt{31}}{2}$

2. $\frac{\sqrt{7}}{2} - i \frac{1}{2}$, $-\frac{\sqrt{7}}{2} - i \frac{1}{2}$

3. $-2 + i$, $-i$

4. $i \frac{\sqrt{5} + 2\sqrt{2}}{3}$, $i \frac{\sqrt{5} - 2\sqrt{2}}{3}$

5. $\frac{\sqrt{2} - \sqrt{3}}{2} + i \frac{\sqrt{6} - 3}{2}$, $-\frac{\sqrt{2} + \sqrt{3}}{2} - i \frac{\sqrt{6} + 3}{2}$

6. $\pm 37^{1/4} \cos\left(\frac{\pi}{2} - \frac{1}{2} \arctan \frac{1}{6}\right) + i \left(\sqrt{6} \pm 37^{1/4} \operatorname{sen}\left(\frac{\pi}{2} - \frac{1}{2} \arctan \frac{1}{6}\right) \right)$

7. $1 - i$, $-1 - 3i$

8. 0 , $\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}$, $-\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}$

9. $\pm 2^{1/4} \exp\left(i \frac{\pi}{8}\right)$, $\pm 10^{1/4} \exp\left(i \frac{1}{2} \arctan 3\right)$

10. -1 , $\frac{1}{2} + i \frac{\sqrt{3}}{2}$, $\frac{1}{2} - i \frac{\sqrt{3}}{2}$, 2 , $-1 + i\sqrt{3}$, $-1 - i\sqrt{3}$

11. $\sqrt{\frac{3}{2}} + i\left(1 + \frac{1}{\sqrt{2}}\right)$, $-\sqrt{\frac{3}{2}} + i\left(1 + \frac{1}{\sqrt{2}}\right)$, $\sqrt{\frac{3}{2}} + i\left(1 - \frac{1}{\sqrt{2}}\right)$, $-\sqrt{\frac{3}{2}} + i\left(1 - \frac{1}{\sqrt{2}}\right)$,
 $i(1 + \sqrt{2})$, $i(1 - \sqrt{2})$

12. 0 , $\pm i \frac{4}{\sqrt{3}}$, $\pm i 4\sqrt{3}$

13. $\sqrt[4]{2} \cos\left(\frac{5}{8}\pi\right) + i\left(-\sqrt{2} + \sqrt[4]{2} \operatorname{sen}\left(\frac{5}{8}\pi\right)\right)$, $-\sqrt[4]{2} \cos\left(\frac{5}{8}\pi\right) + i\left(-\sqrt{2} - \sqrt[4]{2} \operatorname{sen}\left(\frac{5}{8}\pi\right)\right)$,
 $\sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + i\left(-\sqrt{2} + \sqrt[4]{2} \operatorname{sen}\left(\frac{3}{8}\pi\right)\right)$, $-\sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + i\left(-\sqrt{2} - \sqrt[4]{2} \operatorname{sen}\left(\frac{3}{8}\pi\right)\right)$

14. $\frac{6}{41} - i \frac{13}{41}, -\frac{6}{25} - i \frac{17}{25}$

15. $-2i, i \frac{1}{2}, -i \frac{2}{3}$

16. $\pm i(\sqrt{2} + 1), \pm i(\sqrt{2} - 1)$

17. $0, \pm i \frac{\sqrt[4]{3}}{\sqrt{2}}, \pm \left(\frac{3^{1/4}}{2} + i \frac{3^{3/4}}{2} \right)$

18. $\frac{\sqrt{3}}{2} - i \frac{3}{2}, -\frac{\sqrt{3}}{2} - i \frac{1}{2}, 0, -2i, \frac{1}{\sqrt{2}} - i \left(1 + \sqrt{\frac{3}{2}} \right), -\frac{1}{\sqrt{2}} + i \left(-1 + \sqrt{\frac{3}{2}} \right)$

19. $i \frac{\sqrt{5} + 2\sqrt{2}}{3}, i \frac{-\sqrt{5} + 2\sqrt{2}}{3}, i \frac{\sqrt{5} - 2\sqrt{2}}{3}, i \frac{-\sqrt{5} - 2\sqrt{2}}{3}$

20. $\frac{1}{2} + \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + i \left(\frac{3}{2} + \sqrt[4]{2} \operatorname{sen}\left(\frac{3}{8}\pi\right) \right), \frac{1}{2} - \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + i \left(\frac{3}{2} - \sqrt[4]{2} \operatorname{sen}\left(\frac{3}{8}\pi\right) \right),$
 $-\frac{1}{2} + \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + i \left(-\frac{3}{2} + \sqrt[4]{2} \operatorname{sen}\left(\frac{3}{8}\pi\right) \right), -\frac{1}{2} - \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + i \left(-\frac{3}{2} - \sqrt[4]{2} \operatorname{sen}\left(\frac{3}{8}\pi\right) \right)$

E)

1. $e^{-2} \cos 3 + ie^{-2} \operatorname{sen} 3$

3. $e^{-1} \cos 3 + ie^{-1} \operatorname{sen} 3$

2. $e^2 \cos 11 + ie^2 \operatorname{sen} 11$

4. $\cos 8 + i \operatorname{sen} 8$

F)

1. $3e^2 (\cos(-4) + i \operatorname{sen}(-4))$

3. $e^5 (\cos(-12) + i \operatorname{sen}(-12))$

2. $e^6 (\cos(-4) + i \operatorname{sen}(-4))$

4. $\sqrt{2} e^2 \left(\cos\left(1 - \frac{\pi}{4}\right) + i \operatorname{sen}\left(1 - \frac{\pi}{4}\right) \right)$

G)

1. $\log 4 + i \left(2k - \frac{1}{2} \right) \pi, \quad k \in \mathbb{Z}$

2. $\frac{1}{2} \log 13 + i \left((2k + 1)\pi - \arctan \frac{2}{3} \right), \quad k \in \mathbb{Z}$

3. $\left(2k - \frac{1}{4} \right) \pi - i \frac{1}{2} \log 8, \quad k \in \mathbb{Z}$

4. $\frac{2k}{5} \pi + i \frac{4k}{5} \pi, \quad k \in \mathbb{Z}$

5. $\frac{1}{2} \log 5 + i \left(\arctan \frac{1}{2} + (2k + 1)\pi \right), \frac{1}{2} \log 17 + i \left(-\arctan \frac{1}{4} + (2k + 1)\pi \right), \quad k \in \mathbb{Z}$

6. $\frac{2}{3}\pi + 2k\pi - i \log 2$, $-\frac{2}{3}\pi + 2k\pi - i \log 2$, $k \in \mathbb{Z}$
7. $i(2k+1)\pi$, $\frac{1}{2} \log 2 + i\left(\frac{3}{4} + 2k\right)\pi$, $k \in \mathbb{Z}$
8. $\frac{1}{4} \log 32 + i\left(\frac{5}{8} + k\right)\pi$, $k \in \mathbb{Z}$
9. $-\log 2 - i(\pi + 2k\pi)$, $i\left(\frac{2}{3}\pi + 2k\pi\right)$, $i\left(-\frac{2}{3}\pi + 2k\pi\right)$, $k \in \mathbb{Z}$
10. $\pi + 2k\pi$, $\frac{3}{4}\pi + 2k\pi - i\frac{1}{2} \log 2$, $k \in \mathbb{Z}$
11. $\frac{1}{4} \log 2 + \left(k + \frac{1}{8}\right)\pi + \left(-\frac{1}{4} \log 2 + i\left(k + \frac{1}{8}\right)\pi\right)$, $k \in \mathbb{Z}$.
12. $\frac{\pi}{4} + k\pi$, $\frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$