Tuesday, November 8, 2022 11:39 AM

THM (EUCL. CONNECTION THM) $\forall n \in \mathbb{Z}^+$

$\mathbb{R}^n$ EUCLIDEAN IS A CONNECTED SPACE.

THIS IMPLIES : $A \subseteq \mathbb{R}^n$ A NONTRIVIAL ($A \neq \emptyset, \mathbb{R}^n$)

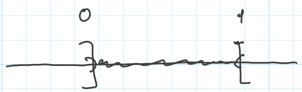
WE HAVE : 1) A OPEN $\Rightarrow$ A NOT CLOSED

2) A CLOSED $\Rightarrow$ A NOT OPEN

INTERESTING CONSEQUENCE :

EXAMPLE / MOTIVATION CONSIDER $]0,1[\subseteq \mathbb{R}$ EUCLIDEAN

$]0,1[$ OPEN SET , $]0,1[\neq \emptyset, \mathbb{R}$.



NOTICE THAT

$0, 1 \notin]0,1[$

ARE ACCUMULATION POINTS !!!

WE CAN GENERALIZE : (X, d) M.SPACE CONNECTED

LET $A \subseteq X$, A OPEN , $A \neq \emptyset, X$. THEN

$\exists x_0 \in X$, $x_0 \notin A$, x_0 ACC. POINT FOR A .

WHY ???

$A \neq \emptyset, X$, A OPEN $\Rightarrow$ HP (X,d) CONNECTED $\Rightarrow$ A IS NOT CLOSED $\Rightarrow$

$\Rightarrow$ IT IS FALSE : A CONTAINS ALL ITS ACC. POINTS $\Rightarrow$

$\Rightarrow$ $\exists x_0 \in X$, $x_0 \notin A$, x_0 ACC. FOR A (OPEN/NONTRIVIAL)

CONT. FUNCTS (X, d_X) , (Y, d_Y) METRIC SPACES

CONSIDER A FUNCT FROM A SUBSET $A \subseteq X$ TO (Y, d_X)

$$F : A \subseteq (X, d_X) \longrightarrow (Y, d_Y)$$

LET $x_0 \in A$. BY DEF:

F CONTINUOUS AT $x_0 \in A$ $\overset{DEF}{\Longleftrightarrow}$ $\forall \varepsilon \in \mathbb{R}^+ \ \exists \delta \in \mathbb{R}^+$

S.T. $d_Y \left(F(x_0), F(x) \right) < \varepsilon$ $(*)$

$\forall x \in I(x_0, \delta) \cap A$

NOTICE THAT. IF $X = Y = \mathbb{R}$ EUCLIDEAN

$(*)$ BECOMES: $\forall \varepsilon \in \mathbb{R}^+ \ \exists \delta \in \mathbb{R}^+$

S.T. $|F(x_0) - F(x)| < \varepsilon$ $(**)$

$\forall x$ IS ST. $|x_0 - x| < \delta$, $x_0 \in A$.

GLOBAL CONTINUITY $F: A \subseteq (X, d_X) \to (Y, d_Y)$.

M.SPACE M.SPACE

F IS CONTINUOUS ON A $\overset{DEF}{\Longleftrightarrow}$ F IS CONTINUOUS AT $x_0 \in A$

$\forall x_0 \in A$.

THM $F: A \subseteq (X, d_X) \to (Y, d_Y)$.

THE FOLLOWING ASSERTIONS ARE EQUIVALENT:

1) F CONTINUOUS ON A

2) $\forall B \subseteq Y$, B OPEN $\Rightarrow \exists B_1 \subseteq X$, B_1 OPEN

SUCH THAT

$$\left(F^{-1}[B] \right) = \underset{\text{OPEN}}{B_1} \cap \underset{\text{TRUE DOMAIN}}{A}$$

REMEMBER 1) $B \subseteq Y$ THE SET $F^{-1}[B]$

DENOTES THE FIBER / PREIMAGE OF B BY THE FUNCT F:

THAT IS

$$F^{-1}[B] \overset{DEF}{=} \{ x \in A ; F(x) \in B \}$$

REMEMBER 2) $F : A \subseteq \mathbb{R} \to \mathbb{R}$

HOW TO PICTURE THE FIBERS:

HOW TO DESCRIBE THE FIBER $F^{-1}[B]$ OF B?

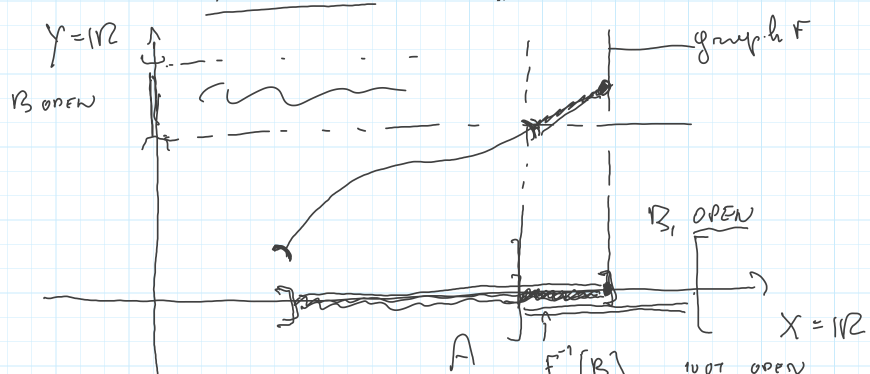
$F^{-1}[B] = \{ x \in A ; F(x) \in B \}$

3) $\forall C \subseteq Y$, C CLOSED $\Rightarrow \exists C_1 \subseteq X$, C_1 CLOSED

S.T. $F^{-1}[C] = C_1 \cap A$

(CLOSED) (DOMAIN)

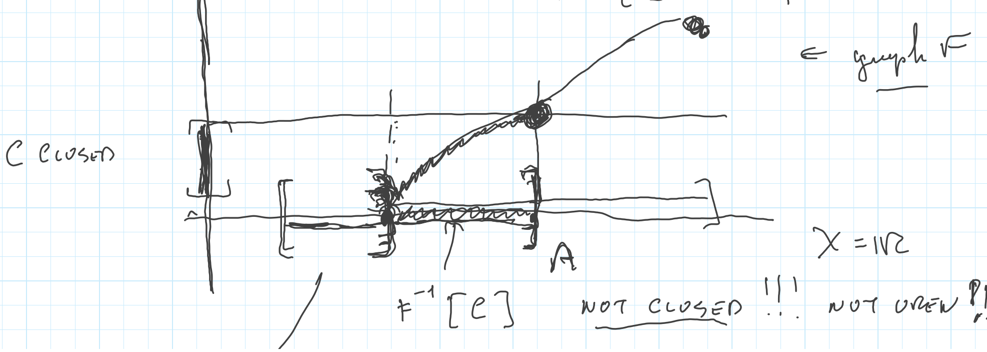
EXAMPLE $F : A \subseteq \mathbb{R}_{EUCL.} \to \mathbb{R}_{EUCL.}$ F CONTINUOUS



NOTICE THAT A NOT CLOSED NOT OPEN

$F^{-1}[B]$ NOT OPEN BUT

AND WE HAVE $F^{-1}[B] = B_1 \cap A$ OK.



$F^{-1}[C]$ NOT CLOSED !!! NOT OPEN ?!!

C_1 CLOSED $\quad F^{-1}[C] = \underset{\text{CLOSED}}{C_1} \cap A.$

BREAK

QUESTIONS??

BEGIN AGAIN AT 12.15