Tuesday, November 8, 2022 11:39 AM

LESSON 4 BIS

ASSERTION SIMPLIFIES: F CONTINUOUS ON A $\Rightarrow$

1) ASSUME $A \subseteq (X, d_X)$ IS OPEN $\overset{THM}{\Rightarrow}$ 2) SAYS:

$\forall B$ OPEN IN $Y \Rightarrow \exists B_1 \subseteq X$, B_1 OPEN S.T.

$$F^{-1}[B] = \underset{\text{OPEN}}{B_1} \cap \underset{\text{OPEN}}{A} \quad \text{OPEN !!!}$$

2) ASSUME $A \subseteq (X, d_X)$ IS CLOSED $\overset{THM}{=}$ 3) SAYS:

$\forall C$ CLOSED IN $Y \Rightarrow \exists C_1 \subseteq X$, C_1 CLOSED S.T.

$$F^{-1}[C] = \underset{\text{CLOSED}}{C_1} \cap \underset{\text{CLOSED}}{A} \quad \text{CLOSED.}$$

3) $A \subseteq (X, d_X)$ IS OPEN/CLOSED $\Rightarrow$ ARE EQUIV:

1) F CONT ON A

2') $\forall B$ OPEN IN $Y \Rightarrow F^{-1}[B]$ IS OPEN

3') $\forall C$ CLOSED IN $Y \Rightarrow F^{-1}[C]$ IS CLOSED

!!!

APPLICATION CONSIDER

$A \subseteq \mathbb{R}^2$ EUCLIDEAN,

$$A = \{ (x,y) \in \mathbb{R}^2 ; \; \underbrace{\overline{107 x^{17} y^3 - 27 x^7 y^{21} + 55 x^8 y^9}}_{?} < 1 \} \; ???$$

> 1

IS A OPEN? YES IS A CLOSED? NO APPL OF THE PRECEDING THM

WHY? WE CAN CONSIDER THE FUNCT

$F : \mathbb{R}^2 \to \mathbb{R}$ WHERE

$$F(x,y) = 107 x^{17} y^3 - 27 x^7 y^{21} + 55 x^8 y^9$$

POLYNOMIAL FUNCT

$\Downarrow$

F CONTINUOUS

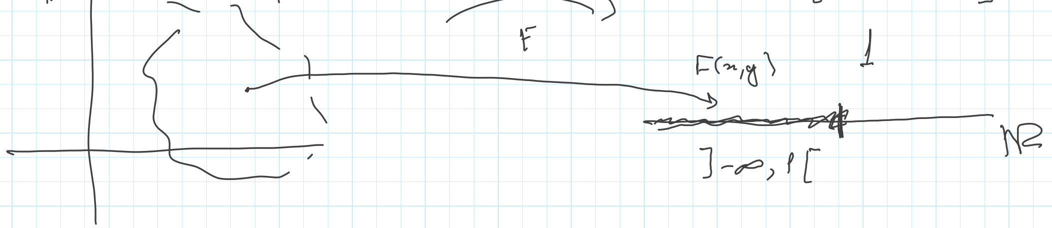
SO, WE CAN REWRITE

$A = \{(x,y) \in \mathbb{R}^2 ;\ F(x,y) < 1\}$ F CONTINUOUS

$\Downarrow$ (>1)

$A = F^{-1}\left[\overset{\text{OPEN}}{]-\infty, 1[}\right] \Rightarrow A$ 2') $\|$ $A = F^{-1}\left[\overset{\text{OPEN}}{]-\infty,1[}\right]$ OPEN

$\Uparrow$ $\Downarrow$ $A = F^{-1}\left[\overset{\text{OPEN}}{]1,+\infty[}\right]$ OPEN



WHAT HAPPENS WEAK INEQUALITY

$B = \{(x,y) \in \mathbb{R}^2 ;\ F(x,y) \leq 1\}$???

≥ 1

$B = F^{-1}\left[\overset{\text{CLOSED}}{]-\infty, 1]}\right] \Rightarrow B$ CLOSED !!!

$\left(B = F^{-1}\left[\underset{\text{CLOSED}}{[1,+\infty[}\right] \right. \Rightarrow B$ CLOSED

SETS DESCRIBED BY EQS ???

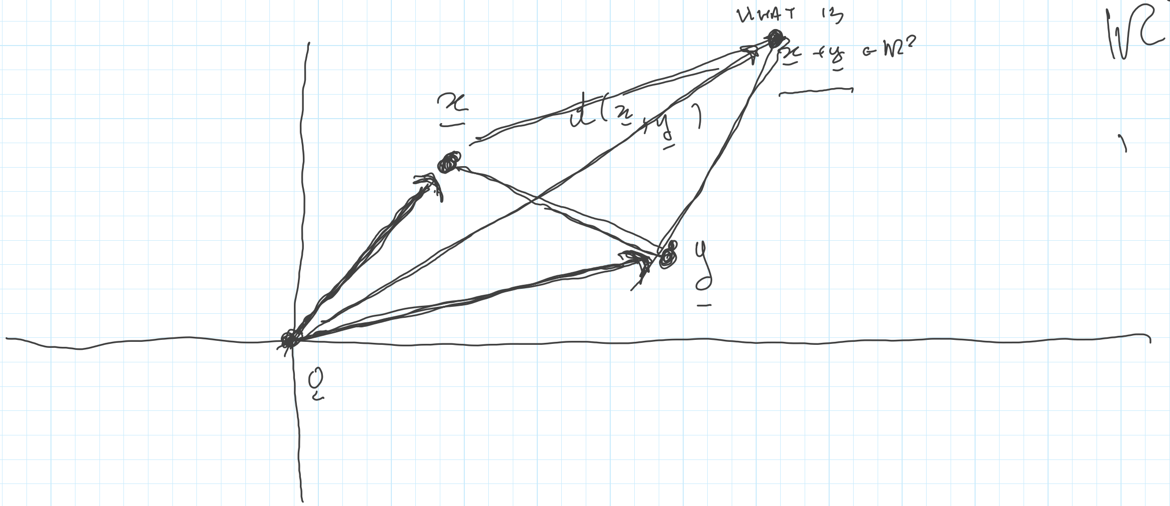
FOR EXAMPLE

$C = \{(x,y) \in \mathbb{R}^2 ;\ x^3y^7 - x^5y^{17} + 13x^3y^3 = 17\}$

SET $F(x,y) = x^3y^7 - x^5y^{17} + 13x^3y^3$, $F: \mathbb{R}^2 \to \mathbb{R}$

$C = F^{-1}\left[\underset{\text{CLOSED}}{\{17\}}\right] \Rightarrow C$ CLOSED

CONSIDER THE EXAMPLE IN $\mathbb{R}^2$

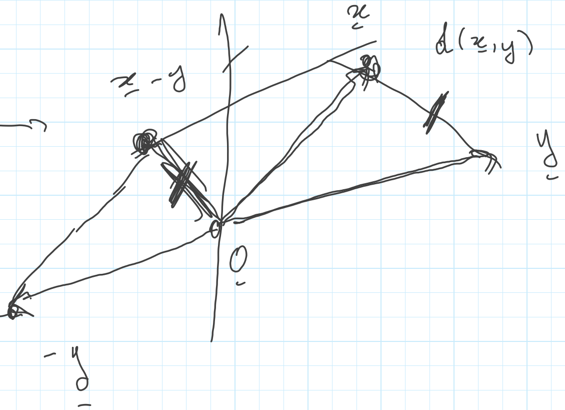


$\mathbb{R}^2$ EUCLIDEAN

BUT NOW

$\underline{x}+(-\underline{y}) =$

$= \underline{x}-\underline{y}$



$-\underline{y}$?

THEN

$d(\underline{x},\underline{y}) =$

$= d(\underline{x}-\underline{y}, \underline{0})$

STOP. QUESTIONS

BYE BYE