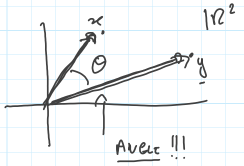
Tuesday, November 8, 2022 11:41 AM

BEGIN AT 15.15

CAN YOU SEE / HEAR ME?

WHAT IS ABOUT ANGLES !???!



$\underline{x}, \underline{y} \neq \underline{0}$

SPACES WITH INNER FORMS (PRODUCTS)

$(X, \langle , \rangle)$, $\langle , \rangle : X \times X \to \mathbb{R}$

X VECTOR SPACE

A SPACE WITH INNER PRODUCT

SUCH THAT

SATISFIES THE FOLLOWING:

i) $\langle \underline{x} + \underline{x}', \underline{y} \rangle = \langle \underline{x}, \underline{y} \rangle + \langle \underline{x}', \underline{y} \rangle$ (1)

AND

$\langle \underline{x}, \underline{y} + \underline{y}' \rangle = \langle \underline{x}, \underline{y} \rangle + \langle \underline{x}, \underline{y}' \rangle$ (1')

WHAT ABOUT THEM?

ii) $\lambda \in \mathbb{R}$,

$\langle \lambda \underline{x}, \underline{y} \rangle = \lambda \langle \underline{x}, \underline{y} \rangle = \langle \underline{x}, \lambda \underline{y} \rangle$ (2) (2')

iii) $\langle \underline{x}, \underline{y} \rangle = \langle \underline{y}, \underline{x} \rangle$ (SYMMETRY)

iv) $\langle \underline{x}, \underline{x} \rangle \geq 0$ & $\langle \underline{x}, \underline{x} \rangle = 0 \iff \underline{x} = \underline{0}$ (ZERO VECTOR)

(THE INNER PRODUCT IS A POSITIVELY DEFINITE QUADRATIC FORM)

RECALL : LINEAR FUNCTS !!!

$F : X \longrightarrow Y$ IS LINEAR $\overset{\text{DEF}}{\Longleftrightarrow}$

VECTOR SPACES

$\overset{\text{DEF}}{\Longleftrightarrow}$ $\begin{cases} F(x_1 + x_2) = F(x_1) + F(x_2) \quad \text{(ADDIVITY)} \\ F(\lambda x) = \lambda \cdot F(x) \quad \lambda \in \mathbb{R} \\ \text{(HOMOGENEITY)} \end{cases}$

THIS MEANS THAT PARTS (1), (2) SAYS THAT THE INNER PRODUCT, REGARDED AS A FUNCT. OF THE LEFT VARIABLE IS LINEAR !!!

BUT (1'), (2') SAYS THAT THE INNER PRODUCT REGARDED AS FUNCT. OF THE RIGHT VARIABLE IS LINEAR !!!

SO, AXIOMS 1) & 2) CAN BE SAID TO BE:

BILINEARITY (TWO TIMES LINEAR, ON THE LEFT AND THE RIGHT, RESPECTIVELY.)

MAIN EXAMPLE (FOR US)

LET $X = \mathbb{R}^n$, $n \in \mathbb{Z}^+$.

GIVEN $\underset{\sim}{x} = (x_1, \dots, x_n)$, $\underset{\sim}{y} = (y_1, \dots, y_n) \in \mathbb{R}^n$

THEN ,

$$\langle \underline{x}, \underline{y} \rangle = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n = \sum_{i=1} x_i y_i \qquad (+)$$

IS AN INNER PRODUCT ON $\mathbb{R}^n$!!!

IT IS CALLED THE

"EUCLIDEAN" (?) INNER PRODUCT ON $\mathbb{R}^n$!!!

WHY, (?).

ANSWER: GENERAL FACT:

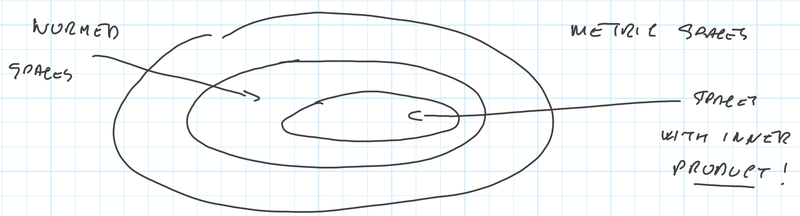
$(X, \langle , \rangle)$ INNER PRODUCT SPACE, AND SET

$\| \; \| : X \to \mathbb{R}$, DEFINED AS:

$\underline{x} \in X \qquad \|\underline{x}\| \overset{\text{DEF}}{=} \sqrt{\langle \underline{x}, \underline{x} \rangle}$, THEN

THIS IS A "NORM FUNCTION" !!!

SO, IN A NAIVE WAY, WE CAN PICTURE!



A QUESTION ARISES ?!?!

IS $\langle \cdot, \cdot \rangle$ EUCLIDEAN PRODUCT ON $\mathbb{R}^n$

A SPECIAL CASE OF INNER PRODUCT ON THEM ???

NO !!! THE "EUCLIDEAN PRODUCT" IS

ESSENTIALLY (?) THE UNIQUE INNER PRODUCT ON $\mathbb{R}^n$!!!

WHAT DOES IT MEAN (!).

SYLVESTER'S LAW OF INERTIA (LINEAR ALGEBRA)

STOP QUESTIONS?

BYE, BYE GOOD WEEKEND