Monday, November 21, 2022 11:06 AM

HELLO!

CAN YOU HEAR / SEE ME?

ANGLES IN A INNER PRODUCT SPACE:

$(X, \langle , \rangle)$ , $\langle , \rangle : X \times X \to \mathbb{R}$

VECTOR SPACE

ST $\langle , \rangle$

1) BILINEAR
2) SYMMETRIC
3) POS. DEFINITE

CONSIDER A SPECIALIZ

EX $X = \mathbb{R}^n$ , $\langle , \rangle$ IS THE EUCLIDEAN INNER PROD.

THAT IS: $\underline{x} = (x_1, \dots, x_n)$ , $\underline{y} = (y_1, \dots, y_n) \in \mathbb{R}^n$

$x_i, y_i \in \mathbb{R}$ AND

$$\langle \underline{x}, \underline{y} \rangle = \sum_{i=1}^{n} x_i y_i \quad \text{(EUCLIDEAN PROD.)}$$

GENERAL SITUATION: HILBERT SPACE

$(\mathbb{R}^n, \langle , \rangle)$ $\langle , \rangle$ EUCLIDEAN PRODUCT.

WE RECALL:

THM (CAUCHY/SCHWARTZ INEQUALITY) WE HAVE:

$\underline{x}, \underline{y} \in \mathbb{R}^n$ ($X$ ANY INNER SPACE)

REMEMBER

$$(*) \quad |\langle \underline{x}, \underline{y} \rangle| \leq \sqrt{\langle \underline{x}, \underline{x} \rangle} \sqrt{\langle \underline{y}, \underline{y} \rangle} = \|\underline{x}\| \cdot \|\underline{y}\|$$

NOW, ASSUME THAT $\underline{x}, \underline{y} \neq \underline{0}$ ; THEN

$(*)$ IMPLIES

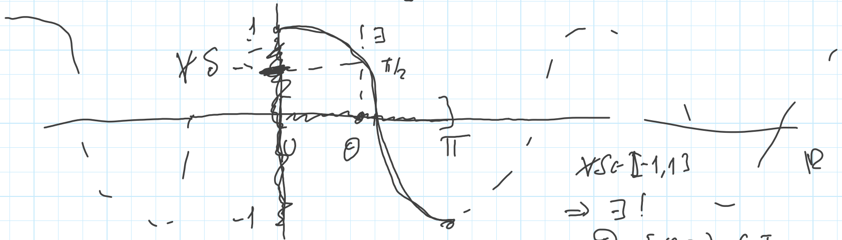
$$|\langle \underline{x}, \underline{y} \rangle|$$

$$(**)\qquad \frac{\phantom{\langle x,y\rangle}}{\underbrace{\|x\|\cdot\|y\|}_{\neq 0}} \le 1$$ , THIS IMPLIES , IN TURN :

$$(***)\qquad -1 \le \underbrace{\frac{\langle x,y\rangle}{\|x\|\cdot\|y\|}}_{\in[-1,1]\ !!!} \le 1 \qquad !!! \ldots$$

NOW , RECALL THE COSINUS FUNCTION

$$\cos(\theta) : [0,\pi] \longrightarrow \mathbb{R} :$$

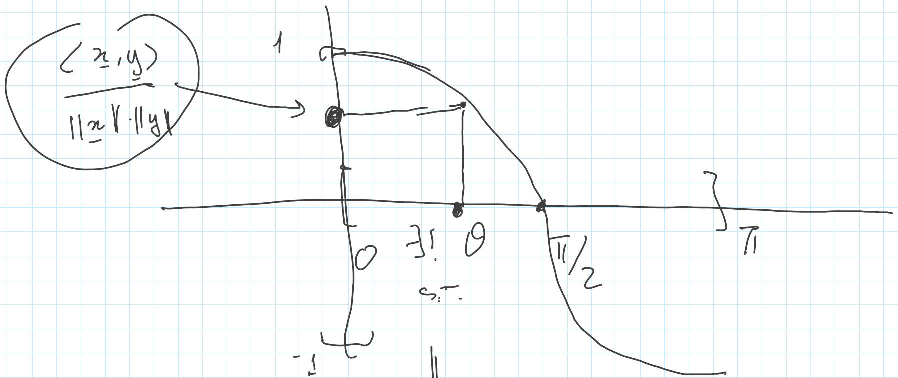



$\forall \delta \in [-1,1] \Rightarrow \exists!\ \theta \in [0,\pi]$ s.t. $\cos(\theta) = \delta$.

IF WE REGARD

$$\cos : [0,\pi] \xrightarrow[1-1]{su} [-1,1]$$

IN PLAIN WORDS , WE HAVE :




$$\Downarrow$$

$\exists!\ \theta \in [0,\pi]$ SUCH THAT

$$\cos(\theta) \overset{!}{=} \frac{\langle x,y\rangle}{\phantom{\|x\|}} \qquad !!!$$

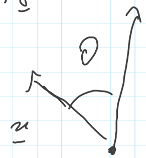
$\|\underline{x}\|\cdot\|\underline{y}\|$ ...

THEN, $\theta$ (UNIQUE) $\in [0,\pi]$ IS THE ANGLE

BETWEEN $\underline{x}, \underline{y} \in \mathbb{R}^n$ ($\underline{x}, \underline{y}$ NONZERO VECTORS!!!).

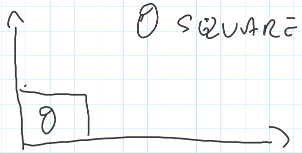
CONCRETE CASES $\underline{x}, \underline{y} \in \mathbb{R}^n$, $\underline{x}, \underline{y} \neq \underline{0}$

1) $\langle \underline{x}, \underline{y} \rangle > 0 \iff$




$0 \leq \theta < \pi/2$ ACUTE

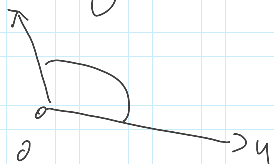
2) $\langle \underline{x}, \underline{y} \rangle = 0 \iff$




$\theta$ SQUARE ANGLE

$\iff \underline{x}, \underline{y}$ ARE ORTHOGONAL.

3) $\langle \underline{x}, \underline{y} \rangle < 0 \iff$




$\theta$ =:

THAT IS

$\pi > \pi/2$

????

BREAK QUESTION?

BEGIN AGAIN AT 15.10