Monday, November 21, 2022 11:13 AM

BEGIN AGAIN AT 12.05

VECTOR VALUED FUNCTIONS (ONE VARIABLE CASE)

THAT IS:

$r : A \subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$,

$t \in A \subseteq \mathbb{R} \quad r : t \longmapsto r(t) \in \mathbb{R}^n$!!!

BUT $r(t) = (r_1(t), \dots, r_n(t)) \in \mathbb{R}^n$

THE SCALAR COMPONENTS OF r

$r_i : A \subseteq \mathbb{R} \longrightarrow \mathbb{R} \quad i = 1, 2, \dots, n$

$r \equiv (r_1, r_2, \dots, r_n)$

VECTOR VALUED

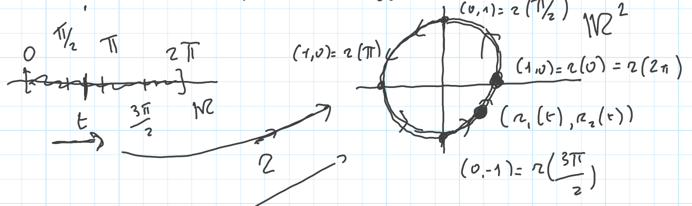
SCALAR COMPONENTS

n-tuple

EX $r : [0, 2\pi] \subseteq \mathbb{R} \longrightarrow \mathbb{R}^2$

WHERE $r(t) = (\cos t, \sin t) \in \mathbb{R}^2 \quad \forall t \in [0, \pi]$

THAT IS $r_1(t) = \cos t$, $r_2(t) = \sin t$.



WHAT IS THE IMAGE

$r[[0, 2\pi]] \stackrel{DEF}{=} \{ r(t) \in \mathbb{R}^2 ; \ t \in [0, 2\pi] \}$???

RECALL

$\cos(t)^2 + \sin(t)^2 = 1$!!!

SATISFIES THE EQ : $x^2 + y^2 = 1$

SO $r[[0, 2\pi]]$ IS THE UNIT CIRCLE IN $\mathbb{R}^2$

WITH CENTER $(0,0)$!!!

WE RECALL $f : B \subseteq \mathbb{R} \longrightarrow \mathbb{R}$

$g : A \subseteq \mathbb{R} \longrightarrow \mathbb{R}$

IF $r[A] \subseteq B$ WE CAN CONSIDER THE COMPOSITION FUNCTION:

$$f \circ r : A \subseteq \mathbb{R} \to \mathbb{R} \quad \text{SUCH THAT}$$

$$(f \circ r)(x) \overset{\text{DEF}}{=} f(r(x)) \quad !!!$$

WE RECALL, THAT GIVEN $a \in A$ AND $b = r(a)$

IF $\exists\, r'(a)$, $\exists\, f'(b)$

THEN $\exists\, (f \circ r)'(a) \overset{\text{THM}}{=} f'(r(a)) \cdot r'(a)$

(THE CHAIN RULE FOR ORDINARY DERIVATIVES !!!)

GENERALIZATION

1) $r : A \subseteq \mathbb{R} \to \mathbb{R}^n$, A OPEN, $a \in A$

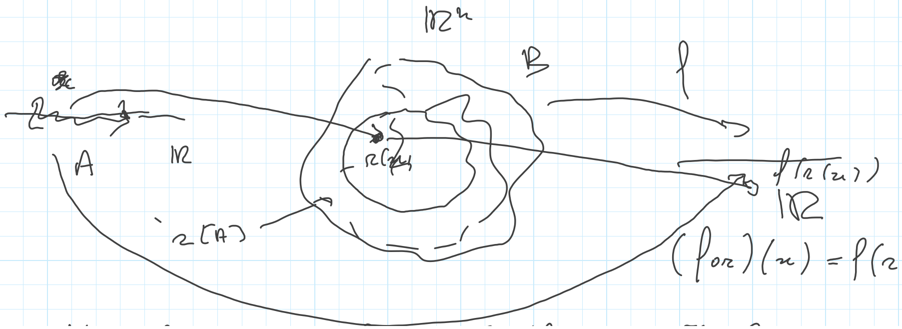
↓ VECTOR VALUED

$r \equiv (r_1, \dots, r_n)$

& SCALAR COMPONENTS

2) $f : B \subseteq \mathbb{R}^n \to \mathbb{R}$, B OPEN, $b \in B$.

ASSUME THAT $r[A] = \{ r(t) \in \mathbb{R}^n ;\ t \in A \} \subseteq B$!!!



$(f \circ r)(x) = f(r(x))$.

WE CAN CONSIDER THE COMPOSITION FUNCT:

$$(f \circ r) : A \subseteq \mathbb{R} \to \mathbb{R} \quad , (f \circ r)(x) = f(r(x)) \; \forall x \in A .$$

THM (REPEAT i) $r[A] \subseteq B$
ii) $b = r(a)$)

HP 1 $\forall i = 1, 2, \ldots, n$

$\exists \, r_i'(a)$ $\left(\exists \; \underline{(r_1'(a), r_2'(a), \ldots, r_n'(a))}\right)$

SCALAR COMPONENTS

HP 2 f DIFFERENTIABLE AT $b = r(a)$

THEN $(f \circ r) : A \subseteq \mathbb{R} \to \mathbb{R}$ (ORDINARY FUNCT.)

TH 1) ADMITS DERIVATIVE $(f \circ r)'(a)$ AT THE POINT $a \in A \subseteq \mathbb{R}$.

TH 2) $\underline{(f \circ r)'(a)} \overset{\text{THM}}{=} \left\langle \overline{\operatorname{grad} f(b)} , \overline{(r_1'(a), \ldots, r_n'(a))} \right\rangle$

?

$\mathbb{R}^n$ $\mathbb{R}^n$

$\left(\operatorname{grad} f(b) = \operatorname{grad} f(r(a)) \right)$

A SPECIAL CASE : $n = 1$

THEN

HP 1) BECOMES 1') $\exists\, r'(a)$

HP 2) BECOMES 2') $\exists\, f'(b)$ (IN PARTICULAR $\text{grad} f(b) = (f'(b))$)

TH 1 BECOMES

$$\exists\, (f \circ r)'(a)$$

TH 2 BECOMES

$$(f \circ r)'(a) \overset{\text{THM}}{=} \langle \text{grad} f(b), (r_1'(a), \dots, r_n'(a)) \rangle$$

BUT $n = 1$

$$\langle (f'(b)), (r'(a)) \rangle = f'(b)\, r'(a)$$

$\uparrow$ $\mathbb{R}$ $\uparrow$ $\mathbb{R}$ $\uparrow$ CHAIN RULE

IS IT CLEAR ???

LOCAL MAX/MIN PNTS FOR FUNCTIONS

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$ (n VARIABLE FUNCTS.)

LET $\underline{x} \in A$. THE POINT $\underline{x} \in A$

IS SAID TO BE A LOCAL MAX/MIN POINT

FOR $f: A \subseteq \mathbb{R}^n \to \mathbb{R}$

IF AND ONLY IF

$\exists\, I(\underline{x}, \delta) \subseteq \mathbb{R}^n$, $\delta \in \mathbb{R}^+$

SUCH THAT

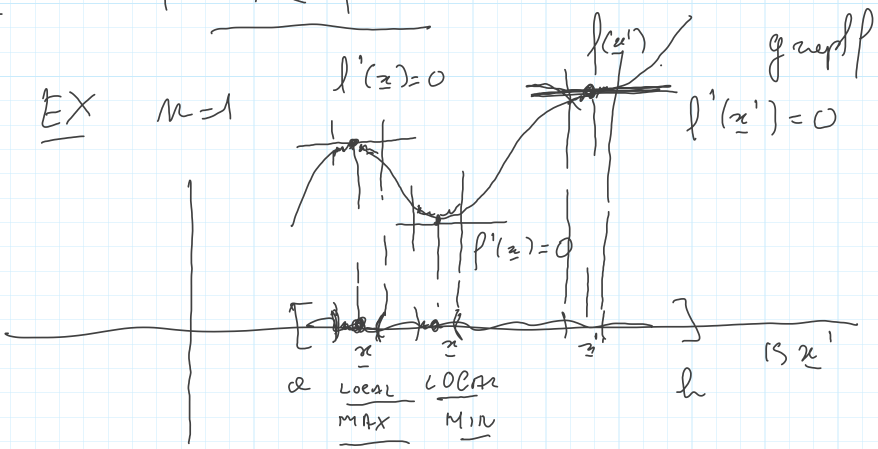
MAX $\quad f(\underline{x}) \geq f(x) \quad \forall\, x \in I(\underline{x},\delta) \cap A$

↓ IF A OPEN

REPLACED BY $x \in I(\underline{x},\delta)$

MIN $\quad f(\underline{x}) \leq f(x) \quad \forall\, x \in I(\underline{x},\delta) \cap A$.

EX $\quad n=1$



BUT $\underline{x}'$ INFLECTION POINT

↓ ↓

IS $\underline{x}'$ LOC MAX / LOC MIN ??? NO

STOP

QUESTIONS?

BYE BYE