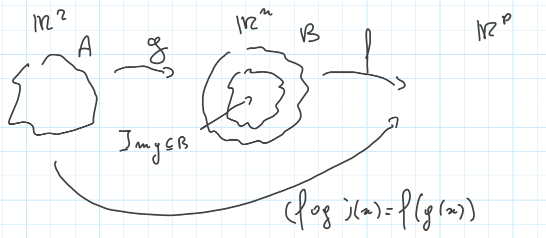
Monday, November 21, 2022 11:17 AM

GENERAL COMP. THM

THM LET $g : A \subseteq \mathbb{R}^n \to \mathbb{R}^m$, A OPEN
AND $f : B \subseteq \mathbb{R}^m \to \mathbb{R}^p$, B OPEN SET.
ASSUME THAT $\operatorname{Im} g = \{ g(x) \in \mathbb{R}^m ; x \in A \} \subseteq B$.

THEN WE CAN CONSIDER THE COMP. FUNCT.:

$$f \circ g : A \subseteq \mathbb{R}^n \to \mathbb{R}^p, \quad (f \circ g)(x) = f(g(x)), \quad \forall x \in A$$



LET $a \in A$, AND $b = g(a)$.

ASSUME THAT:

HPS)
1) g DIFFERENTIABLE AT $a \in A$
2) f DIFFERENTIABLE AT $b = g(a) \in B$.

TH $f \circ g$ DIFFERENTIABLE AT $a \in A$, FURTHERMORE:

$$d(f \circ g)(a) = df(b) \circ dg(a) \qquad (*)$$

$$\begin{tikzcd} \mathbb{R}^n \arrow[r, "dg(a)" ', "\text{LINEAR}"] \arrow[rr, bend right] & \mathbb{R}^m \arrow[r, "df(b)" ', "\text{LINEAR}"] & \mathbb{R}^p \end{tikzcd}$$

$d(f \circ g)(a)$ LINEAR !!!
$d(f \circ g) : \mathbb{R}^n \to \mathbb{R}^p$

IN MATRIX FORM, $(*)$ BECOMES:

$\Rightarrow J \quad = J \times J \qquad (**)$.

$$\underbrace{J_{(f\circ g)(a)}}_{d} \qquad \underbrace{f(b)}_{d} \qquad \underline{g(a)}$$

OF ORDER

$$r \times p = (r \times n) \times (n \times p)$$

SPECIAL CASE (REMEMBER).

$\underline{r=1}$, $p=1$

THEN

$$J_{f(b)} = \left(\frac{\partial f}{\partial x_1}(b), \dots, \frac{\partial f}{\partial x_n}(b)\right) = \text{grad}\, f(x)$$

AND

$$J_{g(a)} = \begin{pmatrix} g_1'(a) \\ \vdots \\ g_n'(a) \end{pmatrix}$$

GENERAL THM

$$J_{(f\circ g)(a)} = J_{f(b)} \times J_{g(a)} =$$

1×1

$$= \left(\frac{\partial f}{\partial x_1}(b), \dots, \frac{\partial f}{\partial x_n}(b)\right) \times \begin{pmatrix} g_1'(a) \\ \vdots \\ g_n'(a) \end{pmatrix} =$$

ROW VECTOR

$$= \left(\langle \text{grad}\, f(b), (g_1'(a), \dots, g_n'(a)) \rangle \right) =$$

$$= \left(\sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(b) \cdot g_i'(a) \right)$$

$$= \left((f \circ g)'(u) \right)$$ 1x1 ORDER MATRIX

THEN, WE RECOVER THE "OLD" COMP. THM. OK???

BREAK QUESTIONS?

BEGIN AGAIN 12.07