Monday, February 6, 2023 11:13 AM

HELLO!!!

BEGIN AT 14.10

QUESTIONS ?

EBESGUE COVERING

LET $A \subseteq \mathbb{R}^n$. A LEB. COV OF A

IS

$\{ I_k ; k \in \alpha \}$ I_k OPEN LIMITED INTERVALS

SUCH THAT

i) α AT MOST COUNTABLE

ii) $A \subseteq \bigcup_k I_k$.

OUTER MEASURE

$A \subseteq \mathbb{R}^n$

FAMILY OF ALL LEB. COV OF A.

$$0 \leq \mu^*(A) \overset{DEF}{=} \inf \left\{ \sum \mu(I_k) ; \{ I_k ; k \in \alpha \} \in \mathfrak{Z}_A \right\}$$

$(\ k \in \ldots)$

0

RMK $\underline{x} \in \mathbb{R}^n$ AND CONSIDER THE SINGLETON SET $\{\underline{x}\}$.

WE HAVE:

$$\mu^*(\{\underline{x}\}) = 0 \quad !!!$$

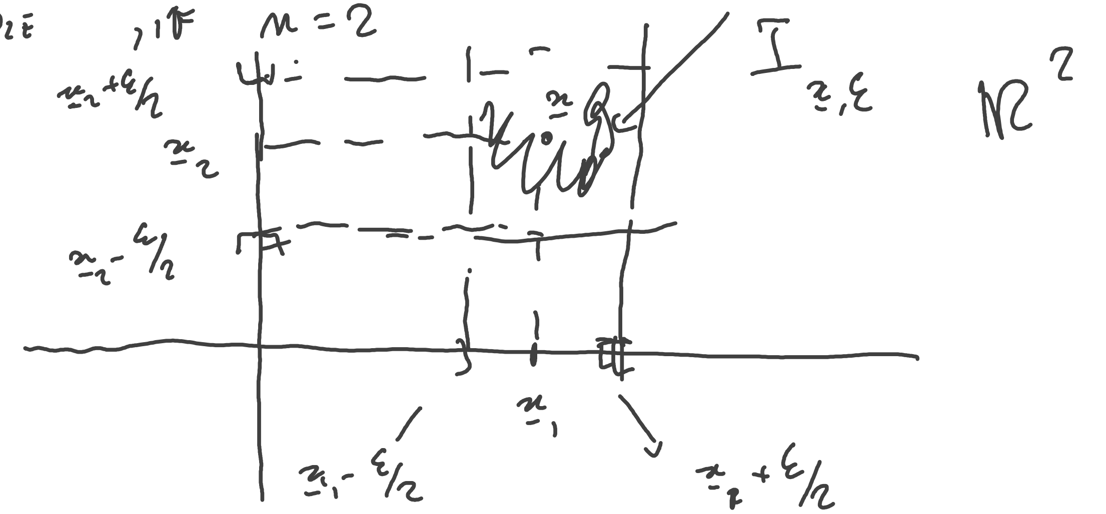
PROOF LET $\varepsilon \in \mathbb{R}^+$ ARBITRARY.

CONSIDER THE OPEN LIMITED INTERVAL:

$$\to I_{\underline{x},\varepsilon} = \left\{ (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n ; \ \underline{x}_i - \varepsilon/2 < x_i < \underline{x}_i + \varepsilon/2, \ i = 1, 2, \ldots, n \right\} \subseteq \mathbb{R}^n$$

WHERE $\underline{x} = (\underline{x}_1, \underline{x}_2, \dots, \underline{x}_n)$.

FOR EXAMPLE, IF $n = 2$



SO $\mu(I_{\underline{x},\varepsilon}) = \varepsilon^n$

NOW, $\underline{x} \in I_{\underline{x},\varepsilon} \Rightarrow \{\underline{x}\} \subseteq I_{\underline{x},\varepsilon} \Rightarrow$

$\Rightarrow \{I_{\underline{x},\varepsilon}\}$ IS A <u>LEB. COV</u> OF $\{\underline{x}\}$. $(*)$

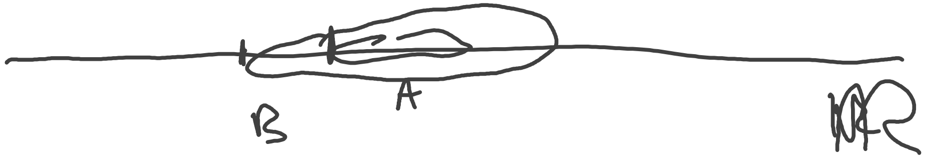
FROM (*) , IT FOLLOWS :

$$\mu^*(\{x\}) \overset{DEF}{=} \inf\left\{\sum_k \mu(I_k);\ \{I_k; k\in a\} \in \mathcal{J}_{\{x\}}\right\}$$

$\leq$ $\qquad$ $\supseteq$

$$\inf\left\{\varepsilon^n = \mu(I_{x,\varepsilon});\ \{I_{x,\varepsilon}\} \in \mathcal{J}_{\{x\}}\right\} =$$

$\inf B \leq \inf A$.



$A \subseteq B$

$$= \inf\{\varepsilon^n;\ \varepsilon\in\mathbb{R}^+\} \Longrightarrow \mu$$

CONSISTENCY PROPERTY

WHAT ABOUT

GIVEN BY DEF

$$\mu^*(I) = \mu(I) \quad , \qquad \text{WHEN}$$

2

$$\overset{\text{MORE}}{=} \mu^*(\overline{I}) \quad , \qquad \text{WHERE}$$

8

$\overline{I}$ IS THE CLOSURE OF

OPEN LIM. INT

REMEMBER GIVEN $A \subseteq \mathbb{R}^n$

IS

CLOSED

$$\overline{A} \overset{DEF}{=} \bigcap C \leftarrow$$

C CLOSED

$C \supseteq A$

SO, $\overline{A}$ IS THE SMALLEST C

THAT CONTAINS A!!

I 

$\bar{I}$ 

MAIN PROPERTIES OF THE OUT

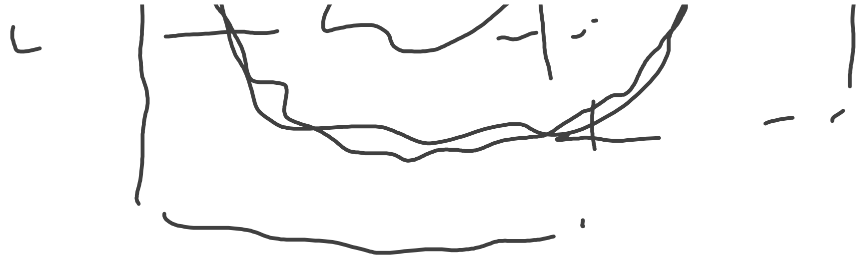
PROP (MONOTONICITY PROPERTY)

$$A \subseteq B \implies \mu^*(A) \leq \mu^*$$

PROOF

NOTICE THAT $A \subseteq B \implies$

$$\{I_k;\ k \in \mathbb{N}\} \in \mathcal{J}_B \implies \{I_k;\ k$$



$$\Longrightarrow \quad \xi_B \subseteq \xi_A$$

now

$$\mu^*(A) = \inf \left\{ \sum_k \mu(I_k) \; ; \; \{ I_k \; ; \; k \right.$$

$$\wedge|$$

$$\cup|$$

$$\mu^*(B) = \inf \left\{ \sum \mu(I) \; \{ I \right.$$

$\mu(B) = \inf \{ \sum_k \mu(I_k) ; \{I_k ; \ldots$

$\Longrightarrow \mu^*(A) \leq \mu^*(\ldots$

CONSEQUENCES

i) CONSIDER $\emptyset \in \mathbb{R}^n$ ..

$$\mu^*(\emptyset) = 0 \quad !!!$$

$\ldots \quad \{x\} \quad \Rightarrow 0 \leq \mu^*(\{\ldots$

$\psi = \{n\} \quad \to n = \mu^*(\psi$

μ

ii) $n = 1$

$$\mu^*(\mathbb{R}) = +\infty$$

CLEARLY $\forall n \in \mathbb{Z}^+$,

$$]0, n[\subseteq \mathbb{R}$$

$$n = \mu(]0,n[) = \mu^{*}(]0,n[) \leq \mu^{*}$$

$$\mu^{*}(\mathbb{R}) \geq n \quad \forall n$$

$$\Downarrow$$

$$\mu^{*}(\mathbb{R}) + ($$

DICEMI QUEST

BEGIN AGAIN

15,12