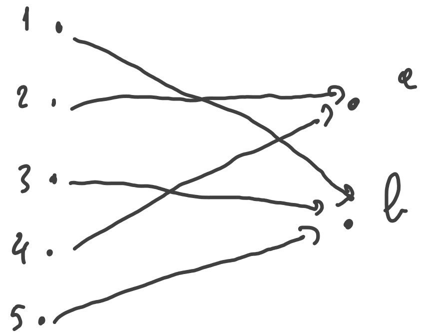
Thursday, May 04, 2023 10:13 AM

INIZIO ORE 10.15

$X = \{1,2,3,4,5\}$, $Y = \{a, b\}$

f



$f(1) = f(3) = f(5) =$

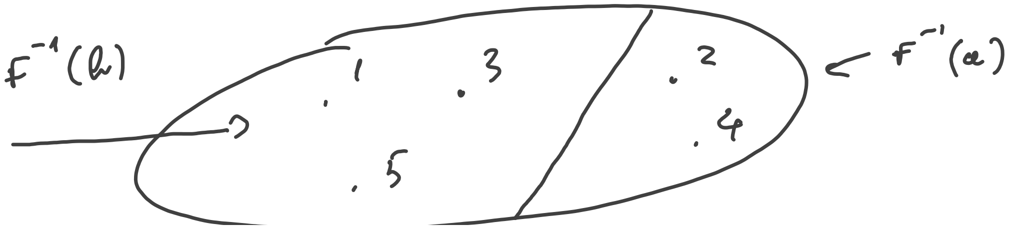
$f(2) = f(4) = a$

$1 \sim_f 3 \sim_f 5 \iff [1]_{\sim_f} = [3]_{\sim_f} = [5]_{\sim_f}$

$2 \sim_f 4 \iff [2]_{\sim_f} = [4]_{\sim_f}$.

Perciò

$\Pi_{\sim_f} = \{ \{1,3,5\}, \{2,4\} \}$ $X = \{1,2,3,4,5\}$



$$\pi : X \to \pi_{\sim_F} = X/_{\sim_F} = \{\{1,3,5\}, \{2,4\}\}$$

OVE $[1]_{\sim_F} = [3]_{\sim_F} = [5]_{\sim_F} = \{1,3,5\}$

$[2]_{\sim_F} = [4]_{\sim_F} = \{2,4\}$

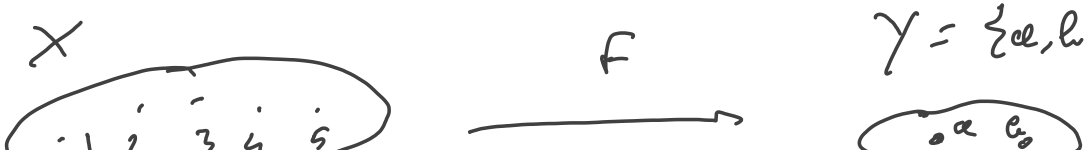
$\overline{F} : \pi_{\sim_F} = X/_{\sim_F} = \{\{1,3,5\},\{2,4\}\}$ t.c.

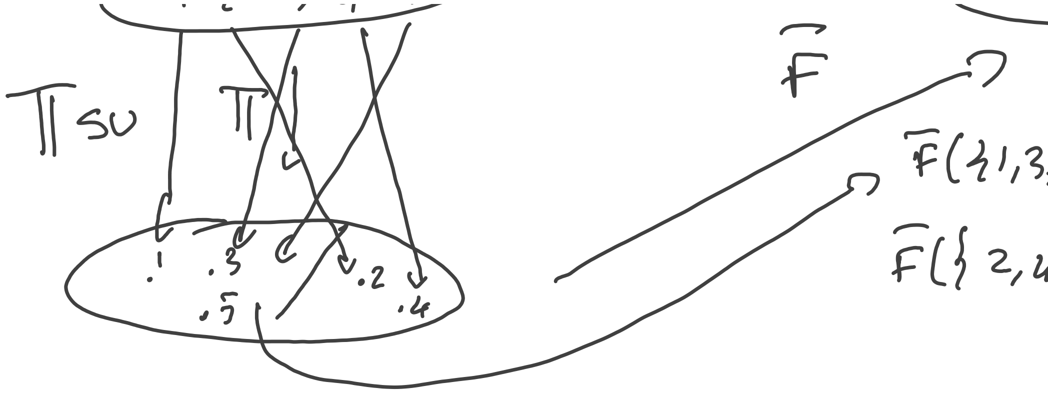
$$\overline{F}([1]_{\sim_F}) = \overline{F}([3]_{\sim_F}) = \overline{F}([5]_{\sim_F}) =$$

$$\overline{F}(\{1,3,5\}) = F(1) = F(3) = F(5) = b$$

ANALOGAMENTE

$$\overline{F}([2]_{\sim_F}) = \overline{F}([4]_{\sim_F}) = \overline{F}(\{2,4\}) \overset{\text{DEF}}{=} F(2) = F(4) = a$$





$\bar{F}(\{1,3,5\}) = b$

$\bar{F}(\{2,4\}) = b$

$\pi_{\sim_F} = X/_{\sim_f}$

CONSEGUENZA

ABBIAMO STABILITO UNA BIIEZIONE TRA

$$\{F : X \to Y\} \quad \& \quad \{(\pi, \bar{F});\ \bar{F} : \pi \xrightarrow{1-1} Y\} \quad !!!$$

π: PARTIZ. X

NE SEGUE

$$\#\{F : X \to Y\} = \#\{(\underset{\text{PART.}}{\pi}, \bar{F});\ \bar{F} : \pi \xrightarrow{1-1} Y\} =$$

$$= \#\left(\dot{\bigcup}\ \{\bar{F} : \pi \xrightarrow{1-1} Y\}\right)$$

π PART DI X

$$= \sum_{\pi \text{ PART DI } X} \# \{ \overline{F} : \pi \xrightarrow{1-1} Y \}$$

!!! (**)

ORA, CASO FINITO, SIA $|X| = n$, $|Y| = m$, $n, m \in \mathbb{N}$.

$$m^n \overset{\text{THM}}{=} \# \{ F : \underline{n} \longrightarrow \underline{m} \} =$$

$$= \sum_{\pi \text{ PART. DI } X = \underline{n}} \# \{ \overline{F} : \pi \xrightarrow{1-1} Y = \underline{m} \} =$$

$$= \sum_{k=0}^{n} \Big(\sum_{\substack{\pi \text{ PART. DI } \underline{n} \\ \text{CON } k\text{-BLOCCHI}}} \# \{ \overline{F} : \underset{\substack{\text{PART} \\ \text{CON } k \text{ BLOCCHI}}}{\pi} \xrightarrow{1-1} Y = \underline{m} \} \Big) =$$

QUANTI SONO GLI ADDENDI? $S(n,k)$

È SEMPRE: $(m)_k$

$$= \sum_{k=0}^{n} S(n,k)\,(m)_k \,.$$

$k=0$

CIOÈ THM $n, m \in \mathbb{N}$

$$(*) \qquad m^n = \sum_{k=0}^{n} S(n,k)\,(m)_k \qquad !!!$$

ORA, SI NOTI CHE

$(+)$

m^n È LA VALUTAZIONE IN m DEL POLINOMIO x^n $(x=m)$

$(m)_k$ È LA VALUTAZIONE IN m DEL POLINOMIO $(x)_k$ $(x=m)$

PERCIÒ I POLINOMI

x^n

k

$$\sum_{k=0}^{n} S'(n,k)\,(x)_k$$

HANNO $\underline{\underline{\text{INFINITE VALUTAZIONI UGUALI}}}$ $(\forall m \in \mathbb{N},\ x = m)$

PERCIÒ L'UNICA IDENTITÀ IN $\mathbb{R}[x]$

$$\underline{x^n = \sum_{k=0}^{n} S'(n,k)\,(x)_k}.$$

EX

$n = 3$

$$S(3,1)(x)_1 + S(3,2)(x)_2 + S'(3,3)(x)_3 = x^3 \quad !!!$$

$\cancel{\longleftrightarrow}$

<u>OK</u> <u>QUESTIONS?</u>