Tuesday, May 16, 2023 10:23 AM

PRINCIPIO INCLUSIONE / ESCLUSIONE

(FORMULA DEL CRIVELLO)

FORMULA DI SYLVESTER

CASO INSIEMISTICO

CARDINALITÀ

DATI DEL PROBLEMA

i) Ω INSIEME FINITO, (SPAZIO CAMPIONE)

ii) $A_1, A_2, \dots, A_n \subseteq \Omega$ ("EVENTI PROIBITI")

iii) SIA $\underline{n} = \{1,2,\dots,n\}$.

CALCOLARE

$$\left|\Omega \setminus \bigcup_{i=1}^{n} A_i\right|$$

VERSIONE PROB.

$$P\left(\Omega \setminus \bigcup_{i=1}^{n} A_i\right)$$

SIANO $S_k := \sum_{T \subseteq \underline{n},\ |T|=k} \left|\bigcap_{i\in T} A_i\right| \quad \forall k = 0,1,\dots,n$ ← NUMERI DI SYLVESTER

THM (SYLVESTER'S FORMULA)

$$\left|\Omega \setminus \bigcup_{i=1}^{n} A_i\right| = \sum_{k=0}^{n} (-1)^k S_k \qquad (*)$$

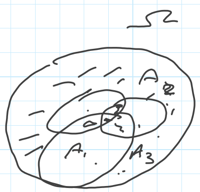
$$\left(= \sum_{T \subseteq \underline{n},\ |T|=k} \left|\bigcap_{i\in T} A_i\right|\right)$$

PER CAPIRE CASO PARTICOLARE DI $(*)$

PER $k = 3$.

Ω SPAZIO CAMPIONE, 3 EVENTI PROIBITI $A_1, A_2, A_3 \subseteq \Omega$

FAMIGLIA DI INDICI: $\underline{3} = \{1,2,3\}$



LA FORMULA $(*)$ DIVIENE:

$$|\Omega \setminus A_1 \cup A_2 \cup A_3| \overset{\text{THM}}{=}$$

$$\underbrace{\Omega}_{k=0} \underbrace{- |A_1| + |A_2| - |A_3|}_{k=1} +$$

$$+ |A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|$$

$k = 2$

$$- |A_1 \cap A_2 \cap A_3|$$

$k = 3$

$\boxtimes$

PRODOTTO COMPLETI

Ω ins, $A_1, A_2, \ldots, A_n \subseteq \Omega$ EVENTI PROIBITI

$\underline{n} = \{1, 2, \ldots n\}$ GLI INDICI

DATO $T \subseteq \underline{n}$ CONSIDERIAMO:

$$(*) \quad \left(\bigcap_{i \in T} A_i \right) \cap \left(\bigcap_{i \notin T} A_i^c \right) \subseteq \Omega$$

PRODOTTI COMPLETI

(*) È DI FATTO L'INSIEME

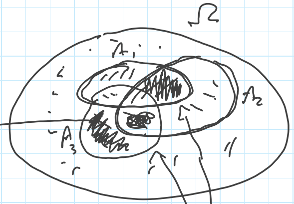
$$\{x \in \Omega \; ; \; x \in A_i \; \forall i \in T \; \& \; x \notin A_i \; \forall i \notin T\} \qquad (**)$$

$n = 3$

EX

$T = \{3\}$

$A_3 \cap A_1^c \cap A_2^c$



$T = \{1, 2\}$

$\rightarrow (A_1 \cap A_2) \cap A_3^c$

$T = \{2, 3\}$

$\rightarrow A_2 \cap A_3 \cap A_1^c$

ORA, FATTO FOND/ELEMENTARE:

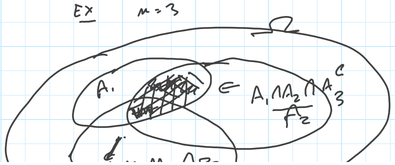
DATO $S \subseteq \underline{n}$

PRODOTTI COMPLETI

$$\bigcap_{i \in S} A_i \stackrel{!}{=} \bigcup_{\substack{T \subseteq \underline{n} \\ T \supseteq S}} \left(\bigcap_{i \in T} A_i \right) \cap \left(\bigcap_{i \notin T} A_i^c \right)$$

$S = \{1, 2\}$

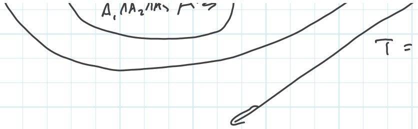
EX $n = 3$



$$\underline{A_1 \cap A_2} = \underline{\bigcap_{i \in S = \{1, 2\}} A_i}$$

$=$

$T \supseteq S = \{1, 2\}$



$T = \{1,2\} \to A_1 \cap A_2 \cap A_3^c$

GRAVRE

$= A_1 \cap A_2 \cap A_3^c \,\dot{\cup}\, A_1 \cap A_2 \cap A_3 \quad T = \{1,2,3\} \to A_1 \cap A_2 \cap A_3$

BREAK QUESTIONS?

BYEBYE