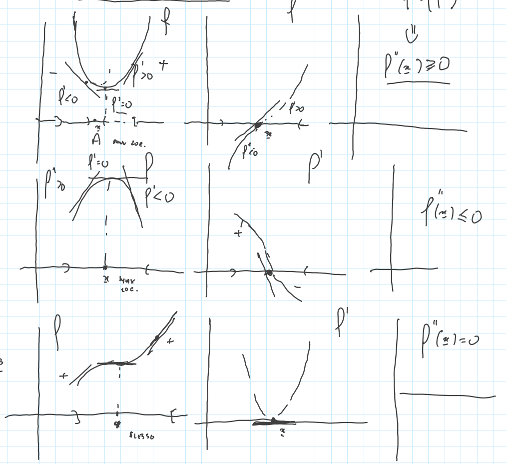
Monday, October 16, 2023 11:29 AM

RICHIAMI PER $n=1$

ES 1

ES 2

ES 3



MAIN CONSTRUCTION

1) $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A APERTO, $\underline{x} \in A$, f DIFF SU A.

2) FISSIAMO UNA DIREZIONE $v \in \mathbb{R}^n$ $(\|v\|=1)$ ("ARBITRARIA")

DEFINISCO

$$r_{\underline{x},v} :]-\delta,\delta[\subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$$ CON δ "SUFFICIENTE PICCOLO"

$$t \in]-\delta,\delta[, \quad r_{\underline{x},v}(t) = \underline{x} + tv \in \mathbb{R}^n$$

$$\shortparallel$$

$$(x_1 + tv_1, \ldots, x_n + tv_n) \quad \forall t$$

ORA

$$r_{\underline{x},v} \equiv (r_{\underline{x},v,1}, \ldots, r_{\underline{x},v,n})$$ OVE

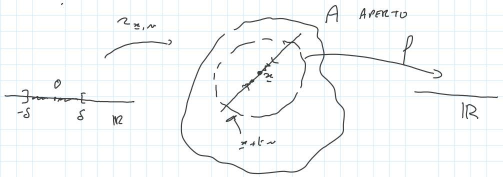
COMP. SCALARI

$i = 1,2,\ldots,n$ $\quad r_{\underline{x},v,i} :]-\delta,\delta[\longrightarrow \mathbb{R}$

$$\forall t \in]-\delta,\delta[\quad r_{\underline{x},v,i}(t) = x_i + tv_i$$

GEOMETRICAMENTE : v VERSORE $\mathbb{R}^n$

$r_{\underline{x},v}(t) = \underline{x} + tv$



"SUFFICIENTE PICCOLO" SIGNIFICA t.c.

$$(*) \quad r_{\underline{x},v}\big[\,]-\delta,\delta[\,\big] = \{ r_{\underline{x},v}(t) \,;\, t \in]-\delta,\delta[\} \subseteq A \text{ APERTO.}$$

DA $(*)$ POSSIAMO FARE LA COMPOSIZIONE :

$$F_{\underline{x},v} = f \circ r_{\underline{x},v} :]-\delta,\delta[\subseteq \mathbb{R} \longrightarrow \mathbb{R}$$

FUNZIONE ORDINARIA

MA ORA $r_{\underline{x},v,i}(t) = x_i + t v_i$ $\qquad i = 1,2,\ldots,n$

$$\Downarrow$$

$(\dagger)$ $\exists\, r'_{\underline{x},v,i}(t) = v_i \quad \forall\, t \in]-\delta,\delta[$

MA ORA f DIFF IN $\underline{x}$. $(\#)$

$(\dagger), (\#)$ APPLICHIAMO IL THM DI COMPOSIZIONE:

$\Rightarrow$ 1) $F_{\underline{x},v} \overset{\text{DEF}}{=} f \circ r_{\underline{x},v}$ È DERIVABILE IN $\underline{0} \in]-\delta,\delta[$

CIOÈ $\exists\; F'_{\underline{x},v}(0) = (f \circ r_{\underline{x},v})'(0)$

E, DI PIÙ,

2) $F'_{\underline{x},v}(0) \overset{\text{THM}}{=} \left\langle \operatorname{grad} f(r_{\underline{x},v}(0)), (r'_{\underline{x},v,1}(0), \dots, r'_{\underline{x},v,n}(0)) \right\rangle$

$\left\langle \operatorname{grad} f(\underline{x}), \underbrace{(v_1, v_2, \dots, v_n)}_{= v} \right\rangle$

$$\boxed{\begin{array}{l} \text{QUINDI} \\ \text{FF !!!} \quad F'_{\underline{x},v}(0) = \langle \operatorname{grad} f(\underline{x}), v \rangle \\ \qquad \text{VERO PER OGNI DIREZIONE } \|v\| = 1 \end{array}}$$