Monday, October 16, 2023 11:32 AM

PERCIÒ

F.F. $F'_{\underline{x},v}(0) \overset{DEF}{=} (f \circ r_{\underline{x},v})'(0) \overset{THM}{=} \langle \operatorname{grad} f(\underline{x}), v \rangle$

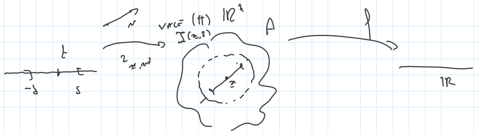
$\forall v \in \mathbb{R}^n, \ \|v\| = 1$

ORA, SIA $\underline{x} \in A$ MAX (MIN) LOCALE PER $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, f DIFF

$\Updownarrow$

$\exists I(\underline{x}, \delta) \subseteq \mathbb{R}^n$ t.c. $f(\underline{x}) \geq f(x)$ $\quad \forall x \in I(\underline{x}, \delta) \quad (\#)$

$(f(\underline{x}) \leq f(x))$



$(\#) \Rightarrow \quad f(\underline{x}) \overset{DEF}{=} f(r_{\underline{x},v}(0)) \geq (\leq) \ f(\underline{x}+tv) \overset{DEF}{=} f(r_{\underline{x},v}(t))$

$\quad \| \qquad\qquad\qquad\qquad \| $

$F_{\underline{x},v}(0) \quad \geq \ (\leq) \quad F_{\underline{x},v}(t) \quad \forall t \in]-\delta, \delta[$

$\Downarrow$

$(\#)$ $0 \in]-\delta, \delta[$ È PTO MAX (MIN) PER $F_{\underline{x},v}$

$\uparrow$ FUNZIONE ORDINARIA

$\Rightarrow F'_{\underline{x},v}(0) \overset{!}{=} 0$

$\overset{F.F.}{=} \langle \operatorname{grad} f(\underline{x}), v \rangle \quad \forall v : \|v\| = 1$

$\Downarrow$

$\langle \operatorname{grad} f(\underline{x}), v \rangle = 0 \quad \forall \|v\| = 1.$

$\Downarrow$ PRENDENDO $v : \underline{e}_1, \underline{e}_2, \dots, \underline{e}_n$ BASE CANONICA

$\operatorname{grad} f(\underline{x}) \overset{DEF}{=} \left(\frac{\partial f}{\partial x_1}(\underline{x}), \dots, \frac{\partial f}{\partial x_n}(\underline{x}) \right) = \underline{0} = (0, \dots, 0) \in \mathbb{R}^n$

CIOÈ

$\frac{\partial f}{\partial x_i}(\underline{x}) = 0 \quad \forall i = 1, 2, \dots, n$

RICORDIAMO f DIFF IN $\underline{x}$. CHI SARÀ $df(\underline{x})$???

ORA, RICORDIAMO, IN GENERALE

$df(\underline{x}) \overset{THM}{=} \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\underline{x}) \cdot dx_i$ $\leftarrow$ FUNZ. LINEARE

($\frac{\partial f}{\partial x_i}(\underline{x})$ = 0 $\leftarrow$ COEFF)

NE SEGUE

THM FOND 1 $\underline{x} \in A \subseteq \mathbb{R}^n$, $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$ DIFF

$\underline{x}$ MAX (MIN) LOCALE PER $f \Rightarrow df(\underline{x}) = \underline{\underline{0}}$ $\leftarrow$ FUNZ. IDENTICAMENTE NULLA, CIOÈ LO ZERO DI $(\mathbb{R}^n)^*$.

SI DICE $\underline{x}$ PTO CRITICO

QUINDI

$\underline{x}$ MAX (MIN) LOCALE PER $f \Rightarrow \underline{x}$ È CRITICO.

(COND NECESSARIA)

$\longrightarrow$

CONTROESEMPIO IN $n=2$

MA NON <u>MIN</u> NÉ <u>MAX</u>.

DIREMO CHE $\underline{x} \in A \subseteq \mathbb{R}^n$, $\underline{x}$ CRITICO, MA

$\underline{x}$ <u>NÉ MIN NÉ MAX</u> È <u>PTO DI SELLA</u>

(<u>SADDLE POINT</u>)

<u>EX "FORMALE"</u> $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^2 - y^2$.

<u>CERCHIAMO I PTI CRITICI:</u>

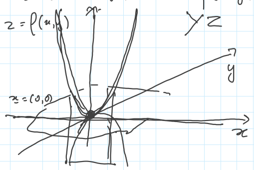
$$\operatorname{grad} f(x,y) \overset{DEF}{=} \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y) \right) =$$

$$= (2x, -2y) \overset{\underline{x} \text{ critico}}{\overset{!}{=}} (0,0) = \underline{0} \in \mathbb{R}^2$$

(x,y) CRITICO $\iff$ $2x = 2y = 0 \Rightarrow x = y = 0 \Rightarrow$

$\exists!$ $\underline{x}$ CRITICO, OVE $\underline{x} = (0,0)$.

CHI È IL GRAFICO DI $f(x,y) = x^2 - y^2$???



STUDIAMO $f(x,y)$ PER $y=0$

$f(x,0) = x^2 \to \operatorname{graph} f \cap XZ$

STUDIAMO $f(x,y)$ PER $x=0$

$f(0,y) = -y^2 \leftarrow \operatorname{graph} f \cap YZ$

LA $f(x,y) = x^2 - y^2$ È <u>PROPRIO</u> LA FUNZ.

CHE DI DÀ LA "SELLA" !!!