Tuesday, November 07, 2023 10:16 AM

(X, d) METRIC SPACE

WE RECALL: $A \subseteq X$

A CLOSED $\overset{\text{DEF}}{\Longleftrightarrow}$ $A^c = X \setminus A$ IS OPEN.

THM (CHARACTERIZATION THM)

A CLOSED $\Leftrightarrow$ A CONTAINS ALL ITS ACC. POINTS

PROOF $\Longrightarrow)$? BY CONTRADICTION.

SUPPOSE $\exists\, x_0 \notin A$, x_0 ACC. POINT FOR A

$\Updownarrow$

$\exists x_0 \in A^c$, x_0 ACC. POINT FOR A

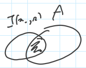
$\Updownarrow$

$\exists x_0 \in A^c$ S.T. $\forall r \in \mathbb{R}^+$ $(I(x_0, r) \setminus \{x_0\}) \cap A \neq \emptyset$

$\boxed{!!!}$

$\Downarrow$

$\forall r \in \mathbb{R}^+$ $I(x_0, r) \cap A \neq \emptyset$



$\Downarrow$

$\forall r \in \mathbb{R}^+$ $I(x_0, r) \nsubseteq A^c$

WE OBTAINED

$\exists\, x_0 \in A^c$ S.T $\forall r \in \mathbb{R}^+$ $I(x_0, r) \nsubseteq A^c$

$\Updownarrow$

$\exists x_0 \in A^c$ S.T. ALL SPHER. NEIGHBOURD

$I(x_0, r) \nsubseteq A^c$

$\Downarrow$

A^c NOT OPEN !!!

$\Downarrow$

A IS NOT CLOSED CONTRADICTION

SO $\Longrightarrow)$ IS PROVED !!!

$\Longleftarrow)$ PROOF BY CONTRADICTION

SUPPOSE A NOT CLOSED $\overset{\text{DEF}}{\Longleftrightarrow}$ A^c NOT OPEN

$\Longleftrightarrow$ $\exists x_0 \in A^c$ S.T. $I(x_0, r) \nsubseteq A^c$ $\forall r \in \mathbb{R}^+$

!

$\Downarrow$

$I(x_0, r) \cap A \neq \emptyset$

$\Downarrow$

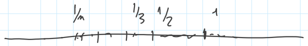
$\exists x_0 \in A^c$ S.T. $\forall r \in \mathbb{R}^+$ $(I(x_0, r) \setminus \{x_0\}) \cap A \neq \emptyset$

$\exists x_0 \in A^c$ S.T. x_0 ACC. POINT FOR A

$\exists\, x_0 \notin A$ CONTRADICTION

SO, $\Leftarrow$) IS PROVEN.

EX $A = \{ 1/m \,;\, m \in \mathbb{Z}^+ \} \subseteq \mathbb{R}$ IS IT CLOSED....??



USING DEF, IS RATHER COMPL. FACT.

BUT WE KNOW 0 IS ACC. POINT FOR A

$$0 \notin A$$

$$\Downarrow$$

$A = \{ 1/m \,;\, m \in \mathbb{Z}^+ \}$ IS NOT CLOSED.

RMK (X, d) METRIC SPACE

$\emptyset \neq A \subseteq X$

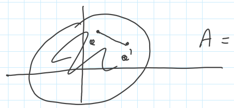
IN PARTICULAR, SET

$d_{/A} : A \times A \longrightarrow \mathbb{R}$ WHERE

$d_{/A}(a, a') = d(a, a')$ $\quad a, a' \in A$

METRIC FUNCT.

$\mathbb{R}^2$



$A = \{ (x, y) \in \mathbb{R}^2 \,;\, x^2 + y^2 \leq 1 \}$

$(A, d_{/A})$ IS METRIC SPACE CALLED METRIC SUBSPACE OF (X, d)

(X, d) M.S, OPEN/CLOSED SUBSETS $A \subseteq X$

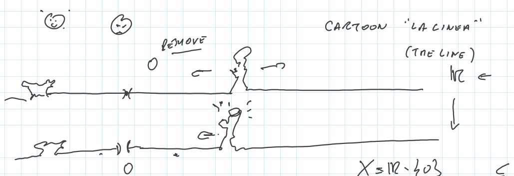
DO THEY EXIST ???

NOTICE THAT $A = X$
1) IT IS OPEN - TRIVIAL
2) IT IS CLOSED - TRIVIAL

1) $\Leftrightarrow$ $\emptyset = X^c$ CLOSED

2) $\Leftrightarrow$ $\emptyset$ OPEN

SO, IN ANY M.S (X, d) THE TRIVIAL SUBSETS : $\emptyset, X$ ARE OPEN/CLOSED !!!



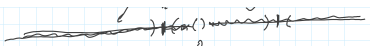
CARTOON "LA LINEA" (THE LINE)

$X = \mathbb{R} \setminus \{0\} \subseteq \mathbb{R}$

$$X = \mathbb{R} \setminus \{0\} = \underbrace{]-\infty, 0[}_{\text{OPEN}} \,\dot\cup\, \underbrace{]0, +\infty[}_{\text{OPEN}}$$

$\neq \emptyset \quad \neq \emptyset$

$\mathbb{R}$



$$X = \mathbb{R} - \{0\} = \overset{\not\swarrow\emptyset}{]-\infty, 0[} \cup \overset{\not\swarrow\emptyset}{]0, +\infty[}$$

OPEN OPEN

BREAK

BYEBYE

<u>QUESTIONS</u> ?