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CONTINUOUS FUNCTION

(X, d_X) , (Y, d_Y) METRIC SPACES

GIVEN $\emptyset \neq A \subseteq (X, d_X)$

CONSIDER A FUNCTION

$$F : A \longrightarrow (Y, d_Y).$$

F IS CONTINUOUS AT THE $\underline{x} \in A$ (LOCAL)

IF AND ONLY IF

(*) $\forall \varepsilon \in \mathbb{R}^+ \quad \exists \delta \in \mathbb{R}^+$ SUCH THAT

$$d_Y (F(x), F(\underline{x})) < \varepsilon \quad \forall x \in I(\underline{x}, \delta) \cap A.$$

NOW SPECIAL CASE $(X, d_X) = (Y, d_Y) = \mathbb{R}$ EUCLIDEAN

(*) BECOMES : $\forall \varepsilon \in \mathbb{R}^+ \; \exists \delta \in \mathbb{R}^+$ SUCH THAT

($**$) $\quad |F(x) - F(\underline{x})| < \varepsilon \quad \forall x \in A, \ |x - \underline{x}| < \delta$

IN GENERAL, WE SAY

$F : A \subseteq (X, d_X) \to (Y, d_Y)$ IS

CONTINUOUS ON A $\iff$

F IS CONTINUOUS AT $\underline{x} \in A$, FOR EVERY $\underline{x} \in A$ (GLOBAL)

THM $\quad F : A \subseteq (X, d_X) \to (Y, d_Y)$

THE FOLLOWING ARE EQUIVALENT:

1) F IS CONTINUOUS ON A

2) $\forall B \subseteq Y$, B OPEN $\Rightarrow \exists B_1 \subseteq X$, B_1 OPEN

SUCH THAT

$$F[B] = B_1 \cap A$$

3) $\forall\, C \subseteq Y$, C $\underline{\text{CLOSED}}$ $\Rightarrow \exists\, C_1 \subseteq X$, C_1 CLOSED

SUCH THAT

$$\underline{F^{-1}[C]} = C_1 \cap A \,.$$

$$F : A \subseteq (X, d_X) \to (Y, d_Y)$$

$B \subseteq Y$ WHAT DOES IT MEAN

$F^{-1}[B]$???...

$B \subseteq Y$

$F^{-1}[B]$