Tuesday, November 14, 2023 7:37 AM

MULTIVARIATE "DIFFERENTIAL CALCULUS"

RECALL, FOR $n = 1$

$f : A \subseteq \mathbb{R} \to \mathbb{R}$, A OPEN, $\underline{x} \in A$.

WE SAY THAT f ADMITS DERIVATIVE AT $\underline{x} \in A$

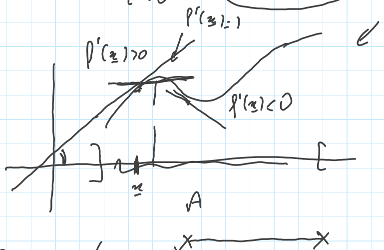
IF AND ONLY IF

INCREMENTAL RATIO

$\exists$ FINITE

$$\lim_{t \to 0} \frac{f(\underline{x}+t) - f(\underline{x})}{t} \overset{\text{DEF}}{=} f'(\underline{x}) \in \mathbb{R}$$

DERIVATIVE OF f AT $\underline{x} \in A$

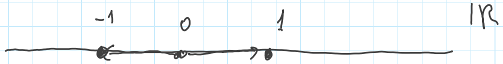


DIRECTION (VERSOR)

$v \in \mathbb{R}^n$, v DIRECTION $\iff \|v\| = 1$.

EX 1 $n = 1$ v DIRECTION IN $\mathbb{R}^1 = \mathbb{R} \iff$

$\|v\| = 1 = |v| \implies v = -1, 1$.

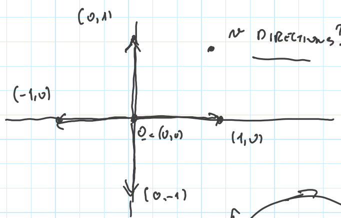


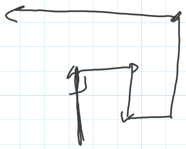
EX $n = 2$ WHAT ARE DIRECTION v IN $\mathbb{R}^2$

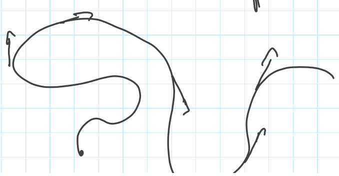
$\Updownarrow$

$\|v\| = 1$

v DIRECTIONS? $\mathbb{R}^2$





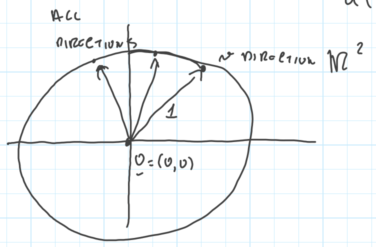


WHY DIRECTIONS v IN $\mathbb{R}^2$ ARE INFINITE ??? ...

RECALL THAT v DIRECTION $\iff$ $\boxed{\|v\| = 1}$

$$\Updownarrow$$

$$d(v, 0) = 1$$



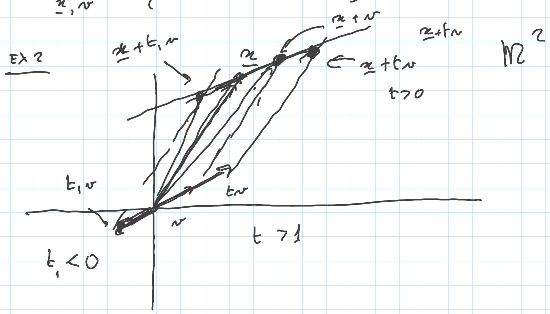
PRELIMINARY REMARK $n \in \mathbb{Z}^+$

FIX $\underline{x} \in \mathbb{R}^n$, v DIRECTION, $\|v\| = 1$

CONSIDER

$$r_{\underline{x},v} = \{ x = \underline{x} + tv;\ t \in \mathbb{R} \} \subseteq \mathbb{R}^n$$

EX 2



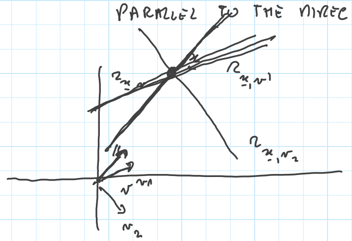
SO $r_{\underline{x},v} \overset{DEF}{=} \{ \underline{x} + tv;\ t \in \mathbb{R} \}$ IS

THE UNIQUE LINE

PASSING THROUGH $\underline{x} \in \mathbb{R}^n$

AND

PARALLEL TO THE DIRECTION v.



LET $n \geq 1$, $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A OPEN, $\underline{x} \in A$.

f ADMITS <u>DIRECTIONAL DERIVATIVE</u>

AT $\underline{x} \in A$ ALONG THE DIRECTION $v : \|v\| = 1$

<u>IF AND ONLY IF</u>

$$\exists \text{ FINITE } \lim_{t \to 0} \frac{\overbrace{f(\underline{x} + tv) - f(\underline{x})}^{\text{INCREMENTAL RATIO}}}{t} \overset{\text{NOT}}{=} \frac{\partial f}{\partial v}(\underline{x}) \qquad (*)$$

IF $n = 1$, THEN $v = 1$ AND

$(*)$ BECOMES

$$\lim_{t \to 0} \frac{f(\underline{x} + t) - f(\underline{x})}{t} = f'(\underline{x})$$