

Corso di Analisi Matematica T-1
Corso di Laurea in Ingegneria dell'Automazione
Anno Accademico 2023/24

Esercizi

A) Calcolare la derivata delle seguenti funzioni:

1. $f(x) = 5x^4 - 2x + 1$

2. $f(x) = \frac{x^2}{x+1}$

3. $f(x) = \frac{x^2}{\sqrt{x^2+2}}$

4. $f(x) = e^x \tan^3 x$

5. $f(x) = x \log x + x^4$

6. $f(x) = e^x \sin x$

7. $f(x) = e^x(\sin x + \cos x)$

8. $f(x) = \frac{\cos x + 1}{x + \sin x}$

9. $f(x) = \frac{\sin(x^2)}{x+1}$

10. $f(x) = \sqrt{1+e^x}$

11. $f(x) = \sin\left(\frac{1}{x^2+x+1}\right)$

12. $f(x) = \frac{\log(2x+3)}{x^2+5}$

13. $f(x) = x \sin(x^5 - 7)$

14. $f(x) = \frac{\cosh(\sinh x)}{\cosh x}$

15. $f(x) = \arctan(\sin(2 \log x))$

16. $f(x) = \frac{\sqrt{x} - 6x^8}{1+3x^5}$

17. $f(x) = e^{x^3 e^x} \cos x$

18. $f(x) = (\sin x)^{x^4}$

19. $f(x) = (x^3 + 4)^{\cos x}$

20. $f(x) = (x^2 + 1)^{x^3+x}$

21. $f(x) = (x^2 + 2)^x$

22. $f(x) = (x^2 + 1)^{\log x}$

B) Calcolare la derivata delle seguenti funzioni nel punto c indicato:

1. $f(x) = 2|x-2| - \log(x^2+x)$, $c = -2$

2. $f(x) = \frac{2 \sin |3x|}{x}$, $c = -\frac{\pi}{2}$

3. $f(x) = \sin(x^3 \log x)$, $c = e$

4. $f(x) = x \log^3\left(\frac{x}{e^2}\right)$, $c = e$

5. $f(x) = \frac{e^{x^2}}{x^5}$, $c = \sqrt{2}$

6. $f(x) = x^2 e^{x^3}$, $c = 2$

7. $f(x) = x^3 \cos(x^4)$, $c = 2$

8. $f(x) = \log|e^{3x^5} - e^3|$, $c = 0$

9. $f(x) = \frac{\exp(3 \sin(2x))}{\sqrt{x^2 - 4x + 16}}$, $c = 4$

10. $f(x) = \arctan(\sqrt{x^2 + 2x + 5})$, $c = 0$

11. $f(x) = \arctan(|x^2 - 4| + 4) \sin x$, $c = 0$

12. $f(x) = (2 + \sin(5x))^{3+\cos(5x)}$, $c = 2\pi$

13. $f(x) = \frac{x}{\sin^2(2x) + \cos(x^2)}, c = \pi$

14. $f(x) = \arctan \frac{(x+4)^2}{x^2+4}, c = 0$

15. $f(x) = \arcsin \left(\frac{\sqrt{x^2-2}}{4} \right), c = 2$

16. $f(x) = \arcsin(|x^2 - 3x| - 2), c = 1$

17. $f(x) = (x^4 + 3x^2)^{\log x}, c = 1$

18. $f(x) = \left(\frac{x+2}{x^3+1} \right)^{x^2}, c = 1$

19. $f(x) = x\sqrt{|x^2-13|^{-4}}, c = 2$

20. $f(x) = \left(\frac{x^2+2}{5x+2} \right)^x, c = 2$

C) Per ciascuna delle seguenti funzioni determinare il dominio naturale, l'insieme dei punti di derivabilità e calcolare la derivata.

1. $f(x) = \frac{e^x}{|x+3|}$

2. $f(x) = \sqrt{x^2+x}$

3. $f(x) = \log(1-|x|) - \cos x$

4. $f(x) = \sqrt{|x+3|+1}$

5. $f(x) = x \exp(|x+2|)$

6. $f(x) = |x-1| \sin|x-1|$

7. $f(x) = \arcsin \frac{x}{x+2}$

8. $f(x) = |x-1| \sin(\pi x)$

9. $f(x) = \sqrt{-x^2-2x}$

10. $f(x) = \exp(|x^2+x|)$

11. $f(x) = \sqrt{3-|x-1|}$

12. $f(x) = \frac{\log|x|-1}{x}$

13. $f(x) = \sqrt{|x+2|-3}$

14. $f(x) = e^{1/x} \sqrt{|x-5|-1}$

Soluzioni

A)

1. $20x^3 - 2$

2. $\frac{x^2 + 2x}{(x+1)^2}$

3. $\frac{x^3 + 4x}{(x^2 + 2)^{3/2}}$

4. $e^x(3 \tan^4 x + \tan^3 x + 3 \tan^2 x)$

5. $\log x + 1 + 4x^3$

6. $e^x(\sin x + \cos x)$

7. $2e^x \cos x$

8. $\frac{-x \sin x - 2 - 2 \cos x}{(x + \sin x)^2}$

9. $\frac{2x(x+1) \cos x^2 - \sin x^2}{(x+1)^2}$

10. $\frac{e^x}{2\sqrt{1+e^x}}$

11. $\cos\left(\frac{1}{x^2+x+1}\right) \frac{2x+1}{(x^2+x+1)^2}$

12. $\frac{2x^2 + 10 - \log(2x+3)(4x^2+6x)}{(2x+3)(x^2+5)^2}$

13. $\sin(x^5 - 7) + 5x^5 \cos(x^5 - 7)$

14. $\frac{\sinh(\sinh x) \cosh^2 x - \cosh(\sinh x) \sinh x}{\cosh^2 x}$

15. $\frac{2 \cos(2 \log x)}{x(1 + \sin^2(2 \log x))}$

16. $\frac{1 - 27x^5 - 96x^7 \sqrt{x} - 108x^{12} \sqrt{x}}{2\sqrt{x}(1 + 3x^5)^2}$

17. $e^{x^3 e^x} (3x^2 e^x \cos x + x^3 e^x \cos x - \sin x)$

18. $(\sin x)^{x^4} \left(\frac{x^4 \cos x}{\sin x} + 4x^3 \log(\sin x) \right)$

19. $(x^3+4)^{\cos x} \left(\frac{3x^2 \cos x}{x^3+4} - \log(x^3+4) \sin x \right)$

20. $(x^2+1)^{x^3+x} (2x^2 + (3x^2+1) \log(x^2+1))$

21. $(x^2+2)^x \left(\log(x^2+2) + \frac{2x^2}{x^2+2} \right)$

22. $(x^2+1)^{\log x} \left(\frac{\log(x^2+1)}{x} + \frac{2x \log x}{x^2+1} \right)$

B)

1. $-\frac{1}{2}$

2. $\frac{8}{\pi^2}$

3. $4e^2 \cos e^3$

4. 2

5. $-\frac{e^2}{8}$

6. $52e^8$

7. $12 \cos 16 - 256 \sin 16$

8. 0

9. $\frac{\exp(3 \sin 8) (48 \cos 8 - 1)}{32}$

10. $\frac{1}{6\sqrt{5}}$

11. $\arctan 8$

12. 160

13. $\frac{\cos(\pi^2) + 2\pi^2 \sin(\pi^2)}{\cos^2(\pi^2)}$

14. $\frac{2}{17}$

15. $\frac{1}{\sqrt{7}}$

16. 1

17. $\log 4$

18. $\frac{3}{2} \left(2 \log \frac{3}{2} - \frac{7}{6} \right)$

19. $-\frac{1}{4} - \frac{1}{3} \log 2$

20. $\frac{1}{4} \left(-\log 2 + \frac{1}{2} \right)$

C)

1. $\text{dom } f = \mathbb{R} \setminus \{-3\}$

$\text{dom } f' = \mathbb{R} \setminus \{-3\}$

$$f'(x) = e^x \frac{(x+2) \operatorname{sgn}(x+3)}{(x+3)^2}$$

2. $\text{dom } f =]-\infty, -1] \cup [0, +\infty[$

$\text{dom } f' =]-\infty, -1[\cup]0, +\infty[$

$$f'(x) = \frac{2x+1}{2\sqrt{x^2+x}}$$

3. $\text{dom } f =]-1, 1[$

$\text{dom } f' =]-1, 0[\cup]0, 1[$

$$f'(x) = \frac{1}{x - \operatorname{sgn}(x)} + \sin x$$

4. $\text{dom } f = \mathbb{R}$

$\text{dom } f' = \mathbb{R} \setminus \{-3\}$

$$f'(x) = \frac{\operatorname{sgn}(x+3)}{2\sqrt{|x+3|+1}}$$

5. $\text{dom } f = \mathbb{R}$

$\text{dom } f' = \mathbb{R} \setminus \{-2\}$

$$f'(x) = \exp(|x+2|) (1 + x \operatorname{sgn}(x+2))$$

6. $\text{dom } f = \mathbb{R}$

$\text{dom } f' = \mathbb{R}$

$$f'(x) = \sin(x-1) + (x-1) \cos(x-1)$$

7. $\text{dom } f = [-1, +\infty[$

$\text{dom } f' =]-1, +\infty[$

$$f'(x) = \frac{1}{|x+2| \sqrt{x+1}}$$

8. $\text{dom } f = \mathbb{R}$

$\text{dom } f' = \mathbb{R}$

$$f'(x) = \operatorname{sgn}(x-1) \sin(\pi x) + \pi|x-1| \cos(\pi x)$$

9. $\text{dom } f = [-2, 0]$

$\text{dom } f' =]-2, 0[$

$$f'(x) = \frac{-x-1}{\sqrt{-x^2-2x}}$$

10. $\text{dom } f = \mathbb{R}$

$\text{dom } f' = \mathbb{R} \setminus \{-1, 0\}$

$$f'(x) = \exp(|x^2+x|) (2x+1) \operatorname{sgn}(x^2+x)$$

11. $\text{dom } f = [-2, 4]$

$\text{dom } f' =]-2, 4[\setminus \{1\}$

$$f'(x) = \frac{-\operatorname{sgn}(x-1)}{2\sqrt{3-|x-1|}}$$

12. $\text{dom } f = \mathbb{R}^*$

$\text{dom } f' = \mathbb{R}^*$

$$f'(x) = \frac{2 - \log|x|}{x^2}$$

13. $\text{dom } f =]-\infty, -5] \cup [1, +\infty[$

$\text{dom } f' =]-\infty, -5[\cup]1, +\infty[$

$$f'(x) = \frac{\operatorname{sgn}(x+2)}{2\sqrt{|x+2|-3}}$$

14. $\text{dom } f =]-\infty, 0[\cup]0, 4] \cup [6, +\infty[$

$\text{dom } f' =]-\infty, 0[\cup]0, 4[\cup]6, +\infty[$

$$f'(x) = e^{1/x} \frac{(x^2 - 2x + 10) \operatorname{sgn}(x-5) + 2}{2x^2 \sqrt{|x-5|-1}}$$