Wednesday, November 15, 2023 12:12 PM

THM (SCHWARZ)

LET $f: A \subseteq \mathbb{R}^n \to \mathbb{R}$, A OPEN, $\underline{x} \in A$. $i, j = 1, 2, \dots, n$, $i \neq j$

HP1 $\exists$ (EXIST) $\dfrac{\partial^{(2)} f}{\partial x_j \partial x_i}$, $\dfrac{\partial^{(2)} f}{\partial x_i \partial x_j}$ OVER

A SUITABLE OPEN SPH. NEIGBR. $I(\underline{x}, \delta)$, $\delta \in \mathbb{R}^+$

HP2 REGARDING: $\dfrac{\partial^{(2)} f}{\partial x_j \partial x_i}$ AND $\dfrac{\partial^{(2)} f}{\partial x_i \partial x_j}$

THEY ARE CONTINUOUS AT $\underline{x} \in A$.

THEN

TH $$\frac{\partial^{(2)} f}{\partial x_j \partial x_i}(\underline{x}) = \frac{\partial^{(2)} f}{\partial x_i \partial x_j}(\underline{x}) \quad (\text{AT } \underline{x} \in A) \; !!!$$

EX $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^3 y + x^2 y^4$

GIVEN ANY $(x,y) \in \mathbb{R}^2$, WE HAVE

$$\frac{\partial f}{\partial x}(x,y) = 3x^2 y + 2x y^4$$

$$\frac{\partial f}{\partial y}(x,y) = x^3 + 4x^2 y^3 \leftarrow$$

$$\frac{\partial^{(2)} f}{\partial y \partial x}(x,y) \overset{DEF}{=} \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}(x,y)\right) = \frac{\partial}{\partial y}\left(3x^2 y + 2x y^4\right)(x,y) =$$

$$= 3x^2 + 8x y^3$$

$$\frac{\partial^{(2)} f}{\partial x \partial y}(x,y) \overset{DEF}{=} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}(x,y)\right) = \frac{\partial}{\partial x}\left(x^3 + 4x^2 y^3\right)$$

$$= 3x^2 + 8x y^3$$

OVER !!! OK

(RECALL) VECTOR VALUED FUNCTS IN ONE VARIABLE

$r : A \subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$

THAT IS

VECTOR VALUED FUNCT

$r : t \in A \subseteq \mathbb{R} \longrightarrow r(t) \in \mathbb{R}^n$

$$(r_1(t), r_2(t), \dots, r_n(t)) \equiv r(t) \in \mathbb{R}^n$$

SCALAR COMPONENTS OF r

WHERE, FOR $i = 1, 2, \dots, n$

$r_i : A \subseteq \mathbb{R} \longrightarrow \mathbb{R}$

$r_i : t \longmapsto r_i(t) \in \mathbb{R}$

EX $\quad n = 2 \qquad$ LET $A = [0, 2\pi] \subseteq \mathbb{R}$

AND

$r : [0, 2\pi] \subseteq \mathbb{R} \longrightarrow \mathbb{R}^2$ WHERE

$r : t \in [0, 2\pi] \longrightarrow r(t) \overset{\text{DEF}}{=} (\cos t, \sin t) \in \mathbb{R}^2$

THAT IS

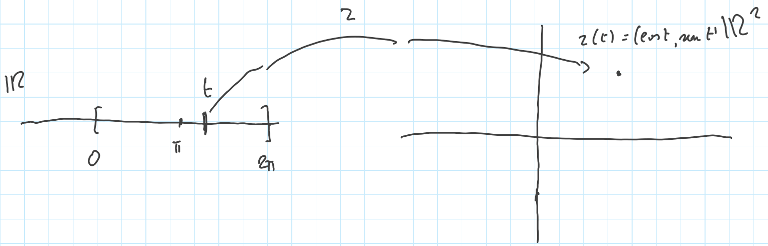
$$r \equiv (r_1, r_2)$$

SCALAR COMPONENTS

$r_1 : [0, 2\pi] \to \mathbb{R}, \quad r_1(t) = \cos t$

$r_2 : [0, 2\pi] \to \mathbb{R}, \quad r_2(t) = \sin t$

OK.



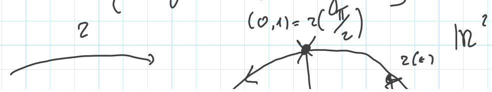
WHAT IS THE IMAGE

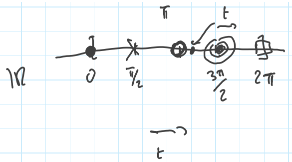
$\operatorname{Im} r \overset{\text{DEF}}{=} \{ r(t) \, ; \, t \in [0, 2\pi] \} =$

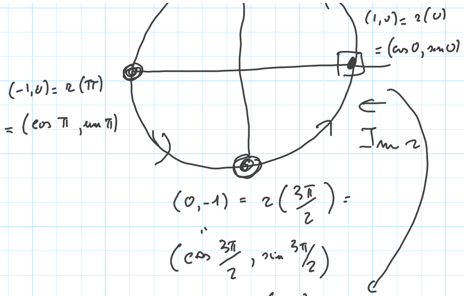
$= \{ (\cos t, \sin t) \in \mathbb{R}^2 \, ; \, t \in [0, 2\pi] \}$

RECALL $\quad (\cos t)^2 + (\sin t)^2 = 1$

THEN $\quad \operatorname{Im} r = \{ (x, y) \in \mathbb{R}^2 \, ; \, x^2 + y^2 = 1 \}$







$$(0,-1) = r\left(\frac{3\pi}{2}\right) = \left(\cos\frac{3\pi}{2}, \sin\frac{3\pi}{2}\right)$$

$$(1,0) = r(2\pi) = (\cos 2\pi, \sin 2\pi).$$

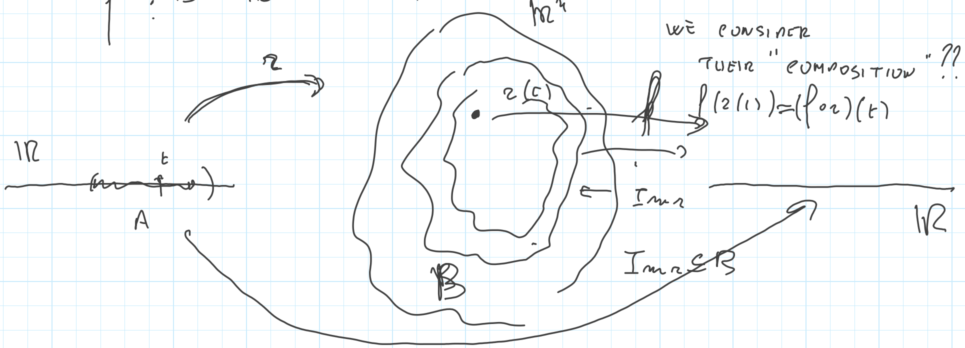
NOW GIVEN

$r : A \subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$ VECTOR VALUED FUNCT.

AND

$f : B \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$ UNDER WHICH CONDS WE CONSIDER THEIR "COMPOSITION" ???

$f(r(t)) = (f \circ r)(t)$



WE CAN CONSIDER THE "COMPOSITION"

$$(f \circ r)(t) = f(r(t)) \qquad \forall t \in A$$

UNDER THE CONDITION $\boxed{\mathrm{Im}\, r \subseteq B}$

NOTICE THAT

$$f \circ r : A \subseteq \mathbb{R} \longrightarrow \mathbb{R}$$

$$(f \circ r)(t) \overset{\text{DEF}}{=} f(r(t)) \qquad \forall t \in A$$

IS AN ORDINARY FUNCTIONS,

THAT IS

A ONE VARIABLE SCALAR VALUED FUNCTION.

BREAK QUESTIONS?

BEGIN AT 10.05