Wednesday, November 15, 2023 12:12 PM

QUESTIONS ?

LET $r : A \subseteq \mathbb{R} \to \mathbb{R}^n$, A OPEN

THAT IS

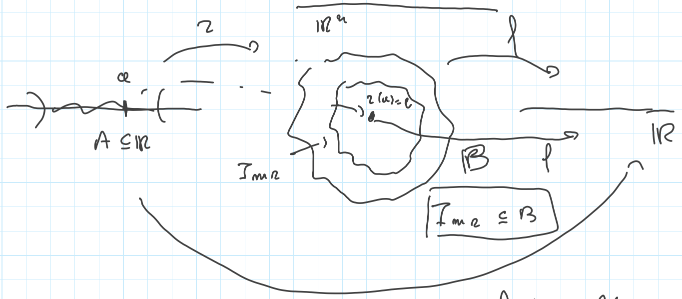
$r \equiv (r_1, r_2, \dots, r_n)$, $r_i : A \subseteq \mathbb{R} \to \mathbb{R}$
$i = 1, 2, \dots, n$

AND

$f : B \subseteq \mathbb{R}^n \to \mathbb{R}$, B OPEN

ASSUME THAT $\boxed{\operatorname{Im} r \subseteq B}$

LET $a \in A$, $b := r(a) \in B \subseteq \mathbb{R}^n$



$f \circ r : A \subseteq \mathbb{R} \to \mathbb{R}$, $(f \circ r)(t) = f(r(t))$ $\forall\, t \in A$

$(f \circ r)(a) = f(r(a)) = f(b)$

ASSUME

Hp 1) THE SCALAR COMPONENT $r_1, r_2, \dots, r_n$ OF r

ADMIT DERIVATIVES AT $a \in A$ THAT IS

$\exists\, (r_1'(a), r_2'(a), \dots, r_n'(a)) \in \mathbb{R}^n$

Hp 2) THE FUNCT f IS DIFFERENTIABLE AT $b = r(a)$

THAT IS

$\exists\, df(b)$!!!

THEN

THESIS THE COMPOSITE FUNCTION

$f \circ r : A \subseteq \mathbb{R} \to \mathbb{R}$

ADMITS DERIVATIVE AT $a \in A$ THAT IS

$$\exists \; (f \circ r)'(a) \qquad (\text{FIRST})$$

MORE (SECOND)

$$(f \circ r)'(a) \overset{\text{THM}}{=} \langle \underbrace{\operatorname{grad} f(b)}_{\mathbb{R}^n}, \underbrace{(r_1'(a), r_2'(a), \dots, r_n'(a))}_{\mathbb{R}^n} \rangle$$

$$(*) \qquad = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(r(a)) \cdot r_i'(a) \qquad \boxed{\begin{matrix} ??? \\ \dots \end{matrix}}$$

WHAT IS THE SPECIAL CASE OF $(*)$ FOR $\boxed{n = 1}$

IN THIS SITUATION, WE HAVE ($n=1$)

$$A \subseteq \mathbb{R} \xrightarrow{r} \operatorname{Im} r \subseteq B \subseteq \mathbb{R} \xrightarrow{f} \mathbb{R}$$

SINCE $n = 1$

$$\Rightarrow f \text{ DIFFERENTIABLE AT } b = r(a) \overset{n=1}{\Longleftrightarrow} \exists \; f'(b)$$

EXISTS THE DERIVATIVE

FROM THE GENERAL ASSERTION, WE HAVE

$$\exists \; (f \circ r)'(a) \overset{\text{THM}}{=} \langle \underset{\substack{\| \\ (f'(b))}}{\operatorname{grad} f(a)}, (r'(a)) \rangle =$$

$$= \langle (f'(b)), (r'(a)) \rangle =$$

$$\overset{\text{TRIVIALLY}}{=} f'(b) \cdot r'(a) = f'(r(a)) \cdot r'(a)$$

THAT IS

$$\boxed{(f \circ r)'(a) = f'(r(a)) \cdot r'(a)}$$ AS WELL-KNOWN

LOCAL MAX/MIN POINTS FOR A FUNCT $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$.

DEF A POINT $\underline{x} \in A$ IS A

LOCAL MAX (MIN) POINT FOR $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$

IF AND ONLY IF

$\exists\ I(\underline{x}, \delta)$, $\delta \in \mathbb{R}^+$ OF $\underline{x} \in A$

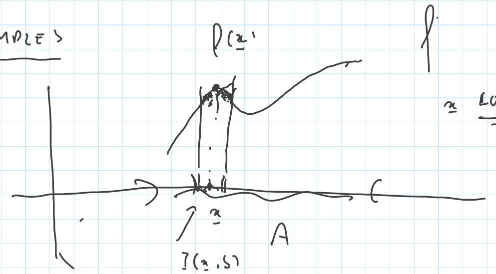
OPEN SPH NEIGH.. SUCH THAT

MAX $f(\underline{x}) \geq f(x) \quad \forall x \in I(\underline{x}, \delta) \cap A$

(MIN $f(\underline{x}) \leq f(x) \quad \forall x \in I(\underline{x}, \delta) \cap A$)

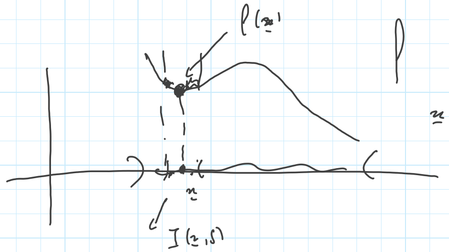
EXAMPLES

1)



$\underline{x}$ LOCAL MAX

2)

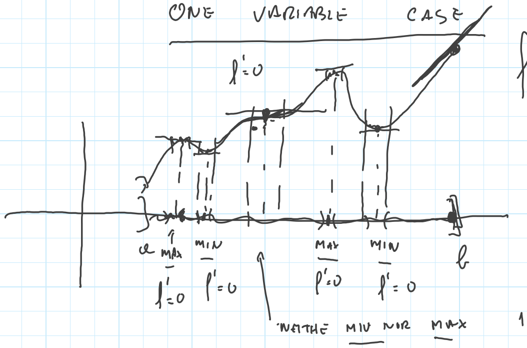


$\underline{x}$ LOCAL MIN.

RECALL SOME FACTS IN THE

ONE VARIABLE CASE

EXAMPLE



$A =]a, b]$ NOT OPEN NOR CLOSED

THEN, WE WILL ASSUME THAT A IS OPEN

THAT IS

$\underline{x}$ LOCAL MAX (MIN) POINT

$\Updownarrow$

$\exists\ I(\underline{x}, \delta) \subset \mathbb{R}^n$ SUCH THAT

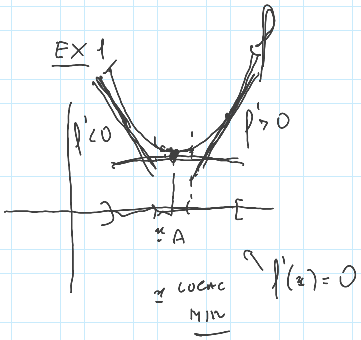
(MAX) $\quad f(\underline{x}) \geq f(x) \quad \forall x \in I(\underline{x}, \delta)$

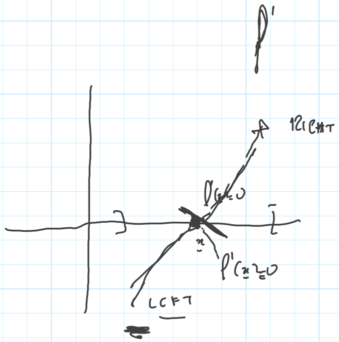
((MIN) $\quad f(\underline{x}) \leq f(x) \quad \forall x \in I(\underline{x}, \delta)$)

GEOMETRIC ANALYSIS OF THE ONE-VARIABLE CASE

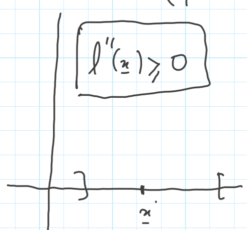
$f'' \stackrel{\text{DEF}}{=} (f')'$

EX 1



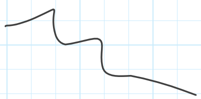


$\boxed{f''(\underline{x}) \geq 0}$



$f''(\underline{x}) < 0$???

ABSURD



IN PLAIN WORDS

$\underline{x}$ LOCAL MIN $\Longrightarrow$

$f'(\underline{x}) = 0 \quad \Longrightarrow \quad f''(\underline{x}) \geq 0$

$$\Rightarrow f(x) = 0 \Rightarrow f(x) \neq 0$$

BYE BYE