Tuesday, December 05, 2023 8:02 AM

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A OPEN, $\underline{x} \in A$, $\underline{f \in C^{(2)}_A}$

NECESSARY CONDITIONS.

<u>THM 1</u> <u>IF</u> $\underline{x} \in A$ MAX (MIN) POINT FOR f

<u>THEN</u> $df(\underline{x}) \equiv \underline{0}$ ($\underline{x}$ <u>CRITICAL</u>)

<u>THM</u> i) $\underline{x} \in A$ <u>MAX</u> point for f $\Longrightarrow$

$\langle v \times H_{f(\underline{x})}, v \rangle \leq 0 \quad \underline{\forall v \in \mathbb{R}^n}$

$\Updownarrow$ <u>LIN. ALG.</u> $\qquad H_{f(\underline{x})}$ NEGATIVELY <u>SEMI</u>DEFINITE

ALL THE n EIGENVALUES $\lambda_1, \lambda_2, \ldots, \lambda_n$ ARE

SUCH $\lambda_i \leq 0 \quad \forall i = 1, 2, \ldots, n$

ii) $\underline{x} \in A$ <u>MIN</u> point for f $\Longrightarrow$

$\langle v \times H_{f(\underline{x})}, v \rangle \geq 0 \quad \forall \underline{v} \in \mathbb{R}^n$

$\Updownarrow$ <u>LIN. ALG</u> $\qquad H_{f(\underline{x})}$ POSITIVELY <u>SEMI</u>DEFINITE

ALL THE n EIGENVALUES $\lambda_1, \lambda_2, \ldots, \lambda_n$ ARE

SUCH THAT $\lambda_i \geq 0 \quad \forall i = 1, 2, \ldots n$

FURTHERMORE, <u>WE HAVE</u> (<u>SUFFICIENT COND.</u>)

<u>THM 3</u> $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$, A OPEN, $\underline{x} \in A$.

IF 1) $\underline{x}$ <u>CRITICAL</u>

and

2 <u>FIRST</u>) $\langle v \times H_{f(\underline{x})}, v \rangle < 0 \quad \underline{\forall v \in \mathbb{R}^n, v \neq \underline{0}}$

$\Updownarrow$ LIN ALG. $\qquad \leftarrow \quad H_{f(\underline{x})}$ NEGATIVELY <u>DEFINITE</u>

ALL THE EIGENVALUES $\lambda_1, \ldots, \lambda_n$ ARE

SUCH THAT $\lambda_i < 0 \quad \forall i = 1, \dots, n$

THEN 1) , 2 FIRST) $\Longrightarrow$ $\underline{x}$ MAX !!!

2 SECOND) $\langle v, H_{f(\underline{x})} v \rangle > 0 \quad \forall v \in \mathbb{R}^n - \underline{0}$

$\Updownarrow$ LIN ALG $\qquad H_{f(x)}$ POSITIVELY DEFINITE

ALL THE EIGENVALUES $\lambda_1, \dots, \lambda_n$ ARE

SUCH THAT $\lambda_i > 0 \quad \forall i = 1, 2, \dots, n$

THEN 1) & 2 SECOND) $\Longrightarrow$ $\underline{x}$ MIN.

THM 1) & 2) IMPLIES THE FOLLOWING

COROLLARY ASSUME THAT

i) $\underline{x}$ CRITICAL

(†) $\rightarrow$ ii) $H_{f(\underline{x})}$ IS NEITHER POSITIVELY SEMIDEFINITE (*)

NOR NEGATIVELY SEMIDEFINITE (**)

THEN $\underline{x} \in A$ IS A SADDLE POINT.

IN FACT (*) $\Longrightarrow$ $\underline{x}$ IS NOT A MIN

(**) $\Longrightarrow$ $\underline{x}$ IS NOT A MAX

WHAT IS A SIMPLE FORM FOR (†)?

IT MEANS THAT THE IS A PAIR

OF EIGENVALUES, SAY λ_i, λ_j $\quad i \neq j$

WITH $\lambda_i > 0$ & $\lambda_j < 0$.