Monday, December 11, 2023 11:34 AM

DINI'S THEOREM

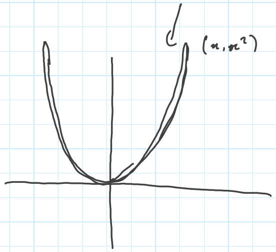
PROBLEM / CONCRETE EXAMPLE 1 IN $\mathbb{R}^2$ (EUCLIDEAN)

LET $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

GRAPH OF f IS

$$\Gamma = \operatorname{graph} f = \{ (x, f(x)) ;\ x \in \mathbb{R} \} = \{ (x, x^2) ;\ x \in \mathbb{R} \}.$$

"PARAMETRIC" DESCRIPTION



BUT Γ CAN BE DESCRIBED ALSO:

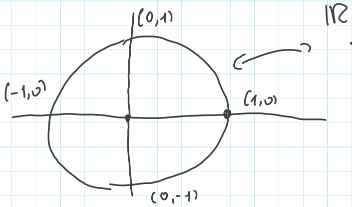
$$\Gamma = \{ (x,y) \in \mathbb{R}^2 ;\ y = x^2 \}$$

THAT IS THE SET OF POINTS SATISFY

$$\boxed{y - x^2 = 0}$$

CARTESIAN DESCRIPTION

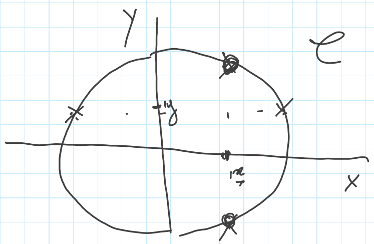
ANOTHER ELEMENTARY EXAMPLE



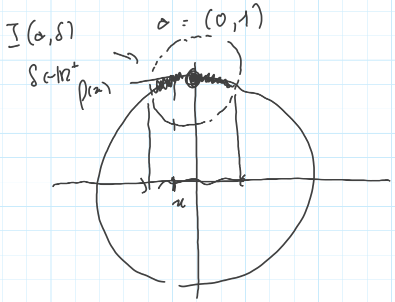
$$\mathcal{C} = \{ (x,y) \in \mathbb{R}^2 ;\ x^2 + y^2 = 1 \}$$

CARTESIAN DESCRIPTION !!!

$\mathcal{C}$ IS THERE A PARAMETRIC DESCRIPTION ? NO !!!



AN IDEA



SHORT DIGRESSION

AT HIGH SCHOOL DID YOU EVER SEEN:

$x^2+y^2=1$

$\Updownarrow$

$$\boxed{\cancel{y=\pm\sqrt{1-x^2}}}$$

NONSENSE!!!

$\mathcal{C}$ LOCALIZE!!! YES

FIX

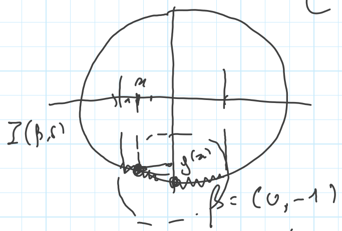
$\alpha=(0,1)$

WE WANT TO PROVIDE A PARAMETRIC DESCR. FOR THE "ARC"

$\mathcal{C}\cap I(\alpha,\delta)$!!!

FOR $\alpha=(0,1)$ THE "ARC"

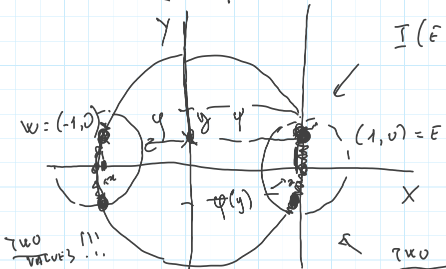
$$\mathcal{C}\cap I(\alpha,\delta)=\left\{(x,f(x))\in\mathbb{R}^2;\ f(x)=+\sqrt{1-x^2}\right\}$$



$$\mathcal{C}\cap I(\beta,\delta)=$$

$$=\left\{(x,g(x))\in\mathbb{R}^2;\ g(x)=-\sqrt{1-x^2}\right\}$$

$I(E,\delta)\cap\mathcal{C}$



TWO VALUES!!!

NO FUNCTION $y=\varphi(x)$!!!

YES, AS FUNCT FROM $y\longrightarrow x=\varphi(y)$. YES

TWO VALUES!!! NO FUNCTION $x\longrightarrow y=\varphi(x)$!!!

$y\longrightarrow\psi(y)=x$,

$x=\psi(y)=+\sqrt{1-y^2}$!!! YES

$\varphi(y) = x = -\sqrt{1-y^2}$

NOTICE THAT

SETTING $F: \mathbb{R}^2 \to \mathbb{R}$, $F(x,y) = x^2 + y^2 - 1$

WE HAVE

$$\mathcal{C} = \{(x,y) \in \mathbb{R}^2;\ x^2+y^2=1\} \overset{\text{TRIVIAL}}{=} \{(x,y) \in \mathbb{R}^2;\ F(x,y) = 0\}$$

AND $F \in C^{(1)}$.

BUT NOW $\alpha = (0,1)$, $\beta = (0,-1)$

$$\frac{\partial F}{\partial x}(x,y) = 2x \Rightarrow \frac{\partial F}{\partial x}(\alpha) = \frac{\partial F}{\partial x}(\beta) = 0 \quad !!!$$

BUT

$$\frac{\partial F}{\partial y}(x,y) = 2y \Rightarrow \underset{!!!}{\underline{\underline{\frac{\partial F}{\partial y}(\alpha) = 2 \neq 0}}},\ \underset{!!!}{\underline{\frac{\partial F}{\partial y}(\beta) = -2 \neq 0}}$$

BUT $\varepsilon = (1,0)$, $w = (0,-1)$

$$\frac{\partial F}{\partial x}(x,y) = 2x \Rightarrow \underset{!!!}{\underline{\underline{\frac{\partial F}{\partial x}(\varepsilon) = 2 \neq 0}}},\ \underset{!!!}{\underline{\underline{\frac{\partial F}{\partial x}(w) = -2 \neq 0}}}$$

WHILE

$$\frac{\partial F}{\partial y}(x,y) = 2y \Rightarrow \frac{\partial F}{\partial y}(\varepsilon) = 0 = \frac{\partial F}{\partial x}(w) \quad !!!$$

BREAK QUESTIONS?

BEGIN AT 15.10