Monday, December 11, 2023 11:35 AM

QUESTIONS ???

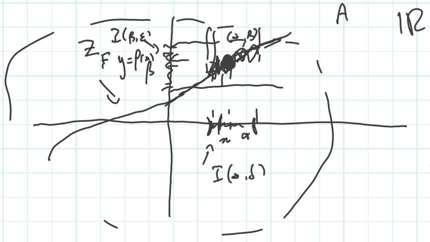
THM (ULISSE DINI ~1915)

LET $F: A \subseteq \mathbb{R}^2 \to \mathbb{R}$, A OPEN, $F \in C^{(1)}_A$.

CONSIDER

$$Z_F = \{(x,y) \in A \in \mathbb{R}^2 ; F(x,y) = 0\}.$$

LET $(\alpha,\beta) \in Z_F$, THAT IS $F(\alpha,\beta) = 0$



ASSUME

$$\frac{\partial F}{\partial y}(\alpha,\beta) \neq 0 \quad !!!$$

THEN, $\exists\ I(\alpha,\delta) \subseteq \mathbb{R}$, $\exists\ I(\beta,\varepsilon) \subseteq \mathbb{R}$

SUCH THAT, FOR ANY $x \in I(\alpha,\delta)$

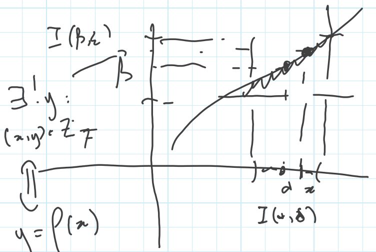
THERE EXISTS UNIQUE $y \in I(\beta,\varepsilon)$ FOR WHICH:

$(x,y) \in Z_F$ (THAT IS $F(x,y) = 0$)

IN PLAIN WORDS, WE CAN EXSPRESS (IN LOCAL SENSE)

THE VARIABLE y AS AN EXPLICIT FUNCTION

$$y = \varphi(x) \quad !!! \quad x \in I(\alpha,\delta)$$



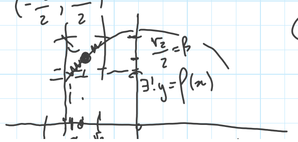
$F: A \subseteq \mathbb{R}^2 \to \mathbb{R}$, A OPEN

$F \in C^{(1)}_A$.

$F(\alpha,\beta) = 0$

$\frac{\partial F}{\partial y}(\alpha,\beta) \neq 0$!!!

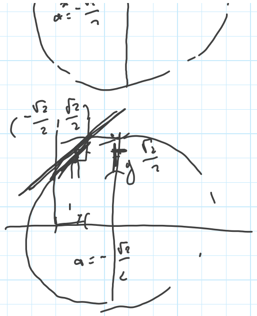
$J = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



$\mathcal{C}$

$$\boxed{x^2 + y^2 = 1}$$

$$\frac{\partial F}{\partial x}(x,y) = 2x \Rightarrow \frac{\partial F}{\partial x}(J) = -\sqrt{2} \neq 0$$



$$f(x) = y = +\sqrt{1-x^2}$$

$$\frac{\partial F}{\partial y}(x,y) = 2y \Rightarrow \frac{\partial F}{\partial y}(y) = \sqrt{2} \neq 0$$

$$\Downarrow$$

$$x = \varphi(y) = -\sqrt{1-y^2}$$

$$F(\alpha,\beta) = 0$$

<u>MORE</u>, SUPPOSE $\frac{\partial F}{\partial y}(\alpha,\beta) \neq 0$

$\exists\ f : I(\alpha,\delta) \longrightarrow I(\beta,\varepsilon)$ SUCH THAT

$y = f(x)$

BUT THE "<u>EXPLICITATING FUNCTION</u>" f IS STILL OF CLASS $C^{(1)}$!!!

AND MORE

$$f'(\alpha) = -\frac{\frac{\partial F}{\partial x}(\alpha,\beta)}{\frac{\partial F}{\partial y}(\alpha,\beta)}$$

$\neq 0$!! <u>NOTICE</u>

<u>GEOMETRIC INTERPRETATION (REGULARITY)</u>

LET $F : \mathbb{R}^2 \to \mathbb{R}$, $F \in C^{(1)}$

A DINI TYPE FUNCTION, THAT IS

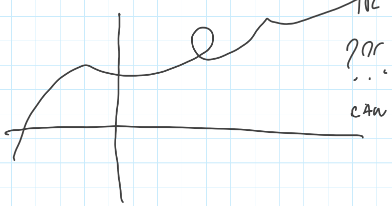
FOR ANY POINT $(\alpha,\beta) \in \mathbb{R}^2$ S.T $\underline{F(\alpha,\beta) = 0}$

<u>EITHER</u> $\frac{\partial F}{\partial x}(\alpha,\beta) \neq 0$ <u>OR</u> $\frac{\partial F}{\partial y}(\alpha,\beta) \neq 0$. !!!

WHAT KIND OF SUBSET OF $\mathbb{R}^2$ IS

$$Z_F \doteq \{(x,y) \in \mathbb{R}^2 ;\ F(x,y) = 0\}.$$

Z_F



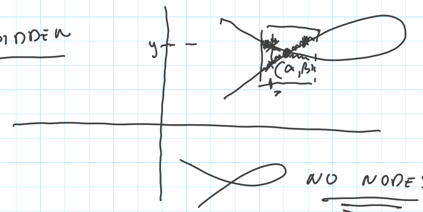
$\mathbb{R}^2$

???

CAN BE <u>ROUGH</u>???

FIRST CAN OCCUR A SITUATIO LIKE THIS?

FORBIDDEN



(α,β) IS A "NODE"

NO EXPLICIT EXPRESSION

$y = f(x)$!

NO NODES

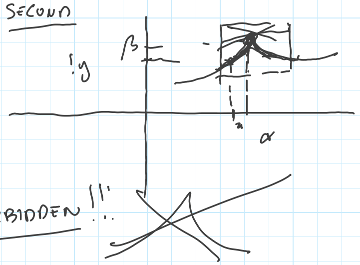
NO EXPLICIT EXSPRESSION !

$x = \varphi(y)$ . .

ABSURD FROM DINI'S THM !!!

Z_F

SECOND



FORBIDDEN !!!

SO, OKAY 'LOCALLY'

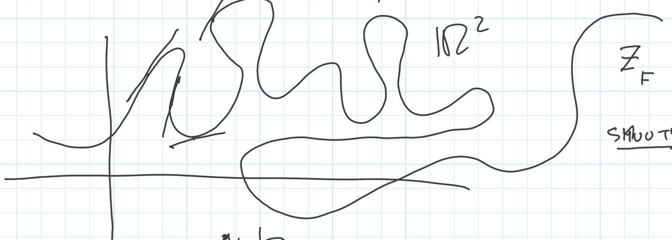
$y = f(x)$

BUT

BUT $\nexists\ f'(\alpha)$!!!

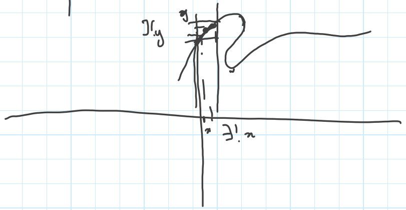
ABSURD FROM DINI'S THEOREM !!!

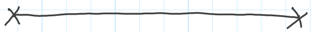
SO, WHAT IS THE GEOMETRIC LOCUS OF ZEROES OF A DINI'S TYPE FUNCTION ???



Z_F

SMOOTH!!





VECTOR VALUED DIFF. FUNCTIONS

LET $D.\ A \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^n$ A OPEN $x \in A$.

THAT IS $\quad f = (f_1, f_2, \dots, f_n)$

SCALAR COMPONENTS

f IS <u>DIFFERENTIABLE</u> AT $\underline{x} \in A$

IF AND ONLY IF

$L_{\underline{x}} : \mathbb{R}^2 \to \mathbb{R}^n$ <u>LINEAR OPERATOR</u>

SUCH THAT

$$(*) \quad \lim_{\underline{h} \to \underline{0} \in \mathbb{R}^2} \frac{f(\underline{x}+\underline{h}) - f(\underline{x}) - L_{\underline{x}}(\underline{h})}{\|\underline{h}\|_2} = \underline{0} \in \mathbb{R}^n$$

BUT $(*)$, WRITTEN INTO SCALAR COMPONENTS BECOMES

$$\begin{cases} \lim_{\underline{h} \to \underline{0}} \dfrac{f_1(\underline{x}+\underline{h}) - f_1(\underline{x}) - L_{\underline{x},1}(\underline{h})}{\|\underline{h}\|} = 0 \in \mathbb{R} \\ \lim_{\underline{h} \to \underline{0}} \dfrac{f_2(\underline{x}+\underline{h}) - f_2(\underline{x}) - L_{\underline{x},2}(\underline{h})}{\|\underline{h}\|} = 0 \in \mathbb{R} \\ \quad - \\ \lim_{\underline{h} \to \underline{0}} \dfrac{f_n(\underline{x}+\underline{h}) - f_n(\underline{x}) - L_{\underline{x},n}(\underline{h})}{\|\underline{h}\|} = 0 \in \mathbb{R}. \end{cases}$$

IN PLAIN WORDS, WE RECOGNIZED THAT

$f : A \subseteq \mathbb{R}^2 \to \mathbb{R}^n$ IS <u>DIFF</u> AT $\underline{x} \in A$ (AS VECTOR VALUED FUNCT)

$$\Updownarrow$$

ALL $f_1, f_2, \dots, f_n$ ARE <u>DIFFERENTIABLE</u> <u>FUNCTS</u> A <u>SCALAR VALUED</u>

SCALAR COMP

THE <u>i-th SCALAR COMPONENT</u> $L_{\underline{x},i}$

OF $L_x : \mathbb{R}^2 \to \mathbb{R}^n$ LINEAR

IS THE DIFFERENTIAL OF THE

i-th SCALAR COMPONENT

f_i OF THE function $f : A \subseteq \mathbb{R}^2 \to \mathbb{R}^n$

BREAK QUESTIONS?