Monday, December 11, 2023 11:37 AM

SO BY HP, WE HAVE

$$
J_{F(y)} = \begin{array}{cccccc} i_1 & i_2 & & & & i_2 \\ \left( x_{i_1} \right. & x_{i_2} & \cdots & \cdots & \cdots & \left. x_{i_2} \right) \\ & & j_1 & j_2 & j_{n-2} & \\ & & y_{j_1} & y_{j_2} & y_{j_{n-2}} & \end{array}
$$

$(n-2)\times n$ MATRIX

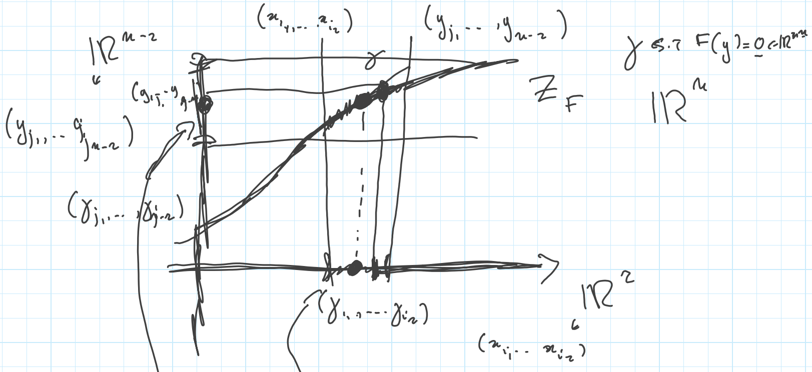
THE CORRESPONDING $(n-2)\times(n-2)$ SQUARE SUBMATRIX HAS DETERMINANT $\neq 0$

SO GIVEN A POINT

$$( \cdot \; - - ) \in \mathbb{R}^n$$
$$=$$
$$( x_{i_1} \cdots y_{j_1} \; \cdots \; y_{j_{n-2}} \; x_{i_2} )$$

$$\mathbb{R}^n = \mathbb{R}^2 \times \mathbb{R}^{n-2}$$

$(x_{i_1},\dots x_{i_2})$ $\quad$ $(y_{j_1},\dots, y_{n-2})$




<u>TH</u> $\quad \exists \; I((\gamma_{i_1}\dots \gamma_{i_2}),\delta) \subseteq \mathbb{R}^2 \qquad \delta\in\mathbb{R}^+$

$\exists \; I((\gamma_{j_1},\dots \gamma_{j_{n-2}}),\varepsilon) \subseteq \mathbb{R}^{n-2} \qquad \varepsilon\in\mathbb{R}^+$

SUCH THAT

$$
(\dagger) \left\{ \begin{array}{l} \forall \, (x_{i_1},\dots, x_{i_2}) \in I((\gamma_{i_1}\dots \gamma_{i_2}),\delta) \\ \exists! \, (y_{j_1},\dots, y_{j_{n-2}}) \text{ s.t.} \end{array} \right.
$$

$$F((x_{i_1}, \dots, x_{i_2}; y_{j_1} \dots y_{j_{m-2}})) = \underline{0} \in \mathbb{R}^{m-2}$$

THAT IS, COND (†) DEFINES A EXPLICIT FUNCTION

$$\varphi : I((y_{i_1} \dots y_{i_2}), \delta) \subseteq \mathbb{R}^2 \longrightarrow J((y_{j_1}, \dots y_{j_{m-2}}), \varepsilon) \subseteq \mathbb{R}^{m-2}$$

EVEN MORE EXPLICITELY, WE CAN WRITE

$$\begin{cases} y_{j_1} = \varphi_{j_1}(x_{i_1}, \dots, x_{i_2}) \quad \text{2 VARIABLES} \\ \quad - \; - \; - \; - \\ y_{j_{m-2}} = \varphi_{j_{m-2}}(x_{i_1}, \dots, x_{i_2}) \end{cases}$$

FURTHERMORE

$$\varphi_{j_1}, \dots, \varphi_{j_{m-2}} \in C^{(1)}_A$$

DONE

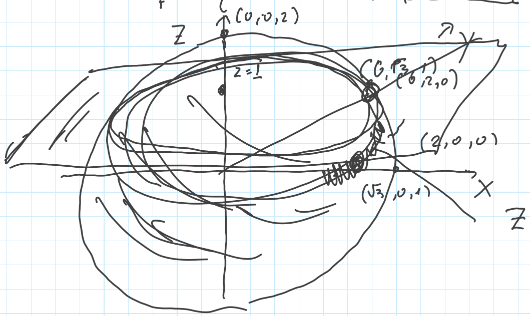
EXAMPLE LET $F : \mathbb{R}^3 \to \mathbb{R}^2$, SO

$F \equiv (F_1, F_2)$ ← 2 SCALAR COMPONENTS WHERE

$$F_1(x, y, z) = x^2 + y^2 + z^2 - 4$$

$$F_2(x, y, z) = z - 1$$

SO $Z_F = \{(x, y, z) \in \mathbb{R}^3 ; \; x^2 + y^2 + z^2 = 4, \; z = 1\}$




$\mathbb{R}^3$

$$x^2 + y^2 + z^2 = 4 \quad (z = 1)$$

$$x^2 + y^2 = 3$$

$$x = \begin{cases} ?\; \boxed{+\sqrt{3 - y^2}} \\ ?\; -\sqrt{3 - y^2} \end{cases}$$

LET $\gamma = (\sqrt{3}, 0, 1) \iff F(\sqrt{3}, 0, 1) = (0, 0) \in \mathbb{R}^2$

$$\Updownarrow$$

$$\gamma = (\sqrt{3}, 0, 1) \in Z_F$$

$$J_{F(x,y,z)} = \begin{pmatrix} 2x & 2y & 2z \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

$$\mathcal{J}_{F(\sqrt{3},0,1)} = \begin{array}{c} \begin{pmatrix} 2\sqrt{3} & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix} \\ \quad y_1 \qquad x_1 \qquad y_2 \end{array}$$ HAS RK = 2

LOCALLY, NEAR BY $\gamma = (\sqrt{3}, 0, 1)$

WE CAN WRITE

$$y_1 = \varphi_1(x_1) = +\sqrt{3 - x_1^2}$$

$$z = y_2 = \varphi_2(x_1) = 1 \quad \text{CONSTANT}$$

NOTICE THAT, BOTH

$$x = y_1 = \varphi_1(\underset{y}{x_1})$$

$$z = y_2 = \varphi_2(\underset{y}{x_1})$$

ARE OF CLASS $\mathcal{C}^{(1)}$ !!!

...

IF WE CONSIDER THE POINT

$$\gamma' = (0, -\sqrt{3}, 1) \iff F(0, -\sqrt{3}, 1) = \underline{0} \in \mathbb{R}^2$$

$$\Updownarrow$$

$$\gamma' \in Z_F$$

$$\mathcal{J}_{F(\gamma')} = \begin{array}{c} \begin{pmatrix} 0 & -2\sqrt{3} & 2 \\ 0 & 0 & 1 \end{pmatrix} \\ x_1 = x \quad y_1 = y \quad y_2 = z \end{array}$$

$rk(\mathcal{J}_{F(\gamma')}) = 2$ MAX

$$y = y_1 = \varphi_1(\underset{x}{x_1}) = -\sqrt{3 - x_1} = -\sqrt{3 - x}$$

$$z = y_2 = \varphi_2(\underset{x}{x_1}) = 1 \quad \text{CONST} \quad = 1 \quad \text{COST.}$$

BREAK / STOP QUESTIONS ?