

**Corso di Analisi Matematica T-1**  
Corso di Laurea in Ingegneria dell'Automazione  
Anno Accademico 2023/24

## Esercizi

A) Scrivere in forma algebrica i seguenti numeri complessi:

1.  $\frac{3-i}{4-i}$                       2.  $\frac{2-i}{2+i}$                       3.  $\frac{4-3i}{(2+i)^2}$                       4.  $\frac{(2\sqrt{3}+i)^3}{\sqrt{3}-i}$

B) Scrivere in forma trigonometrica i seguenti numeri complessi:

1.  $\sqrt{3}-i$                       6.  $(-\sqrt{3}+i)^7$                       10.  $\frac{1-i}{(\sqrt{3}-i)^4}$   
2.  $\frac{1}{-1+\sqrt{3}i}$                       7.  $(1+4i)^5$                       11.  $\frac{(1+i)^5}{(1-i\sqrt{3})^3}$   
3.  $-1-3i$                       8.  $(-1-2i)^6$                       12.  $\frac{(1+2i)(1+i)^8}{(4i)^2}$   
4.  $-1+3i$   
5.  $\frac{-1+2i}{4i}$

C) Determinare le radici quadrate e cubiche dei seguenti numeri complessi:

1.  $-3$                       2.  $-i$                       3.  $1-i\sqrt{3}$                       4.  $1-i$                       5.  $-1+2i$                       6.  $2+i$

D) Risolvere le seguenti equazioni in campo complesso:

1.  $z^2 + z + 8 = 0$                       13.  $(z+4)^6 = (z-4)^6$   
2.  $z^2 + iz - 2 = 0$                       14.  $(z^2 + i2\sqrt{2}z - 1)^2 = -1$   
3.  $z^2 + 2z + 1 + 2i = 0$                       15.  $\left(\frac{z-i}{2z+i}\right)^2 = 8i$   
4.  $(3+3i)z^2 + \sqrt{5}(2-2i)z + 1+i = 0$                       16.  $\left(\frac{z^2+3iz}{z^2+2}\right)^2 = 1$   
5.  $2z^2 + 2(\sqrt{3}+3i)z - 1 + \sqrt{3}i = 0$                       17.  $\left(z + \frac{1}{z}\right)^2 = (1-i)^4$   
6.  $z^2 - i2\sqrt{6}z - i = 0$                       18.  $\left(z^2 + 2iz - \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 = 1$   
7.  $iz^2 - 4z + 2 - 4i = 0$                       19.  $\left((3+3i)z + \frac{1+i}{z}\right)^2 = 5(1+i)^6$   
8.  $z^3 + iz = 0$                       20.  $\left(2iz - \frac{1+2i}{z}\right)^2 = (6-2i)^2$   
9.  $z^4 - 4z^2 + 4 + 2i = 0$   
10.  $z^6 - 7z^3 - 8 = 0$   
11.  $(z-i)^6 = -8$   
12.  $\left(z^2 + \frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^3 = -i$

**E)** Scrivere in forma algebrica i seguenti numeri complessi:

1.  $e^{-2+3i}$                       2.  $\exp((2+i)^3)$                       3.  $\frac{e^{2+i}}{e^{3-2i}}$                       4.  $\exp((1-i)^6)$

**F)** Scrivere in forma trigonometrica i seguenti numeri complessi:

1.  $3e^{2-4i}$                       2.  $(e^{3-2i})^2$                       3.  $e^{(3-2i)^2}$                       4.  $(1-i)e^{2+i}$

**G)** Risolvere le seguenti equazioni in campo complesso:

1.  $e^z = -4i$                       5.  $e^{2z} + 6e^z + 9 + 2i = 0$                       9.  $(e^{-z} + 1)^3 = -1$   
2.  $e^z = -3 + 2i$                       6.  $e^{iz} + 4e^{-iz} = -2$                       10.  $e^{iz} + (1-i)e^{-iz} - i + 2 = 0$   
3.  $e^{iz} = 2 - 2i$                       7.  $e^z + e^{-z} = ie^{-z} + i - 2$                       11.  $e^{(1+i)z} = 1 + i$   
4.  $e^{(2+i)z} = 1$                       8.  $(e^{2z} + 4)^2 = (ie^{2z} - 4)^2$                       12.  $e^{4iz} + (1-i)e^{2iz} - i = 0$

## Soluzioni

A)

1.  $\frac{13}{17} - \frac{1}{17}i$

3.  $-i$

2.  $\frac{3}{5} - \frac{4}{5}i$

4.  $\frac{19}{4} + \frac{53\sqrt{3}}{4}i$

B)

1.  $2\left(\cos\left(\frac{11}{6}\pi\right) + \sin\left(\frac{11}{6}\pi\right)i\right)$

2.  $\frac{1}{2}\left(\cos\left(-\frac{2}{3}\pi\right) + \sin\left(-\frac{2}{3}\pi\right)i\right)$

3.  $\sqrt{10}\left(\cos(\arctan 3 + \pi) + \sin(\arctan 3 + \pi)i\right)$

4.  $\sqrt{10}\left(\cos(\pi - \arctan 3) + \sin(\pi - \arctan 3)i\right)$

5.  $\frac{\sqrt{5}}{4}\left(\cos\left(\arctan \frac{1}{2}\right) + \sin\left(\arctan \frac{1}{2}\right)i\right)$

6.  $128\left(\cos\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{\pi}{6}\right)i\right)$

7.  $17^{5/2}\left(\cos(5 \arctan 4) + \sin(5 \arctan 4)i\right)$

8.  $5^3\left(\cos(6 \arctan 2 + \pi) + \sin(6 \arctan 2 + \pi)i\right)$

9.  $64(\cos 0 + \sin 0 i)$

10.  $\frac{1}{8\sqrt{2}}\left(\cos\left(\frac{5}{12}\pi\right) + \sin\left(\frac{5}{12}\pi\right)i\right)$

11.  $\frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)i\right)$

12.  $\sqrt{5}\left(\cos(\arctan 2 + \pi) + \sin(\arctan 2 + \pi)i\right)$

C)

1.  $\pm\sqrt{3}i$ ;  $\frac{\sqrt[3]{3}}{2} + \frac{3^{5/6}}{2}i$ ,  $-\sqrt[3]{3}$ ,  $\frac{\sqrt[3]{3}}{2} - \frac{3^{5/6}}{2}i$

2.  $\pm\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$ ;  $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ ,  $i$ ,  $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$

3.  $\pm\left(\sqrt{\frac{3}{2}} - \frac{1}{\sqrt{2}}i\right)$ ;  $\sqrt[3]{2} \exp\left(\frac{5+6k}{9}\pi i\right)$ ,  $k = 0, 1, 2$

4.  $\pm 2^{1/4} \left( \cos\left(\frac{7}{8}\pi\right) + \sin\left(\frac{7}{8}\pi\right) i \right);$   
 $2^{1/6} \left( \cos\left(\frac{7}{12}\pi\right) + \sin\left(\frac{7}{12}\pi\right) i \right), -\frac{1}{\sqrt[3]{2}} - \frac{1}{\sqrt[3]{2}} i, 2^{1/6} \left( \cos\left(\frac{23}{12}\pi\right) + \sin\left(\frac{23}{12}\pi\right) i \right)$
5.  $\pm 5^{1/4} \exp\left(\frac{1}{2}(\pi - \arctan 2)i\right); 5^{1/6} \exp\left(\frac{1}{3}((2k+1)\pi - \arctan 2)i\right), k = 0, 1, 2$
6.  $\pm 5^{1/4} \exp\left(\frac{1}{2} \arctan \frac{1}{2} i\right); 5^{1/6} \exp\left(\frac{1}{3} \left(\arctan \frac{1}{2} + 2k\pi\right) i\right), k = 0, 1, 2$

D)

1.  $-\frac{1}{2} - \frac{\sqrt{31}}{2} i, -\frac{1}{2} + \frac{\sqrt{31}}{2} i$
2.  $\frac{\sqrt{7}}{2} - \frac{1}{2} i, -\frac{\sqrt{7}}{2} - \frac{1}{2} i$
3.  $-2 + i, -i$
4.  $\frac{\sqrt{5} + 2\sqrt{2}}{3} i, \frac{\sqrt{5} - 2\sqrt{2}}{3} i$
5.  $\frac{\sqrt{2} - \sqrt{3}}{2} + \frac{\sqrt{6} - 3}{2} i, -\frac{\sqrt{2} + \sqrt{3}}{2} - \frac{\sqrt{6} + 3}{2} i$
6.  $\pm 37^{1/4} \cos\left(\frac{\pi}{2} - \frac{1}{2} \arctan \frac{1}{6}\right) + \left(\sqrt{6} \pm 37^{1/4} \sin\left(\frac{\pi}{2} - \frac{1}{2} \arctan \frac{1}{6}\right)\right) i$
7.  $1 - i, -1 - 3i$
8.  $0, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$
9.  $\pm 2^{1/4} \exp\left(\frac{\pi}{8} i\right), \pm 10^{1/4} \exp\left(\frac{1}{2} \arctan 3 i\right)$
10.  $-1, \frac{1}{2} + \frac{\sqrt{3}}{2} i, \frac{1}{2} - \frac{\sqrt{3}}{2} i, 2, -1 + \sqrt{3} i, -1 - \sqrt{3} i$
11.  $\sqrt{\frac{3}{2}} + \left(1 + \frac{1}{\sqrt{2}}\right) i, -\sqrt{\frac{3}{2}} + \left(1 + \frac{1}{\sqrt{2}}\right) i, \sqrt{\frac{3}{2}} + \left(1 - \frac{1}{\sqrt{2}}\right) i, -\sqrt{\frac{3}{2}} + \left(1 - \frac{1}{\sqrt{2}}\right) i,$   
 $(1 + \sqrt{2})i, (1 - \sqrt{2})i$
12.  $0, \pm \frac{\sqrt[4]{3}}{\sqrt{2}} i, \pm \left(\frac{3^{1/4}}{2} + \frac{3^{3/4}}{2} i\right)$
13.  $0, \pm \frac{4}{\sqrt{3}} i, \pm 4\sqrt{3} i$

$$14. \sqrt[4]{2} \cos\left(\frac{5}{8}\pi\right) + \left(-\sqrt{2} + \sqrt[4]{2} \sin\left(\frac{5}{8}\pi\right)\right)i, \quad -\sqrt[4]{2} \cos\left(\frac{5}{8}\pi\right) + \left(-\sqrt{2} - \sqrt[4]{2} \sin\left(\frac{5}{8}\pi\right)\right)i, \\ \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + \left(-\sqrt{2} + \sqrt[4]{2} \sin\left(\frac{3}{8}\pi\right)\right)i, \quad -\sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + \left(-\sqrt{2} - \sqrt[4]{2} \sin\left(\frac{3}{8}\pi\right)\right)i$$

$$15. \frac{6}{41} - \frac{13}{41}i, \quad -\frac{6}{25} - \frac{17}{25}i$$

$$16. -2i, \quad \frac{1}{2}i, \quad -\frac{2}{3}i$$

$$17. \pm(\sqrt{2} + 1)i, \quad \pm(\sqrt{2} - 1)i$$

$$18. \frac{\sqrt{3}}{2} - \frac{3}{2}i, \quad -\frac{\sqrt{3}}{2} - \frac{1}{2}i, \quad 0, \quad -2i, \quad \frac{1}{\sqrt{2}} - \left(1 + \sqrt{\frac{3}{2}}\right)i, \quad -\frac{1}{\sqrt{2}} + \left(-1 + \sqrt{\frac{3}{2}}\right)i$$

$$19. \frac{\sqrt{5} + 2\sqrt{2}}{3}i, \quad \frac{-\sqrt{5} + 2\sqrt{2}}{3}i, \quad \frac{\sqrt{5} - 2\sqrt{2}}{3}i, \quad \frac{-\sqrt{5} - 2\sqrt{2}}{3}i$$

$$20. \frac{1}{2} + \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + \left(\frac{3}{2} + \sqrt[4]{2} \sin\left(\frac{3}{8}\pi\right)\right)i, \quad \frac{1}{2} - \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + \left(\frac{3}{2} - \sqrt[4]{2} \sin\left(\frac{3}{8}\pi\right)\right)i, \\ -\frac{1}{2} + \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + \left(-\frac{3}{2} + \sqrt[4]{2} \sin\left(\frac{3}{8}\pi\right)\right)i, \quad -\frac{1}{2} - \sqrt[4]{2} \cos\left(\frac{3}{8}\pi\right) + \left(-\frac{3}{2} - \sqrt[4]{2} \sin\left(\frac{3}{8}\pi\right)\right)i$$

**E)**

$$1. e^{-2} \cos 3 + e^{-2} \sin 3 i$$

$$3. e^{-1} \cos 3 + e^{-1} \sin 3 i$$

$$2. e^2 \cos 11 + e^2 \sin 11 i$$

$$4. \cos 8 + \sin 8 i$$

**F)**

$$1. 3e^2(\cos(-4) + \sin(-4)i)$$

$$3. e^5(\cos(-12) + \sin(-12)i)$$

$$2. e^6(\cos(-4) + \sin(-4)i)$$

$$4. \sqrt{2}e^2\left(\cos\left(1 - \frac{\pi}{4}\right) + \sin\left(1 - \frac{\pi}{4}\right)i\right)$$

**G)**

$$1. \log 4 + \left(2k - \frac{1}{2}\right)\pi i, \quad k \in \mathbb{Z}$$

$$2. \frac{1}{2} \log 13 + \left((2k + 1)\pi - \arctan \frac{2}{3}\right)i, \quad k \in \mathbb{Z}$$

$$3. \left(2k - \frac{1}{4}\right)\pi - \frac{1}{2} \log 8 i, \quad k \in \mathbb{Z}$$

$$4. \frac{2k}{5} \pi + \frac{4k}{5} \pi i, \quad k \in \mathbb{Z}$$

5.  $\frac{1}{2} \log 5 + \left( \arctan \frac{1}{2} + (2k+1)\pi \right) i, \frac{1}{2} \log 17 + \left( -\arctan \frac{1}{4} + (2k+1)\pi \right) i, \quad k \in \mathbb{Z}$

6.  $\frac{2}{3} \pi + 2k\pi - \log 2 i, -\frac{2}{3} \pi + 2k\pi - \log 2 i, \quad k \in \mathbb{Z}$

7.  $(2k+1)\pi i, \frac{1}{2} \log 2 + \left( \frac{3}{4} + 2k \right) \pi i, \quad k \in \mathbb{Z}$

8.  $\frac{1}{4} \log 32 + \left( \frac{5}{8} + k \right) \pi i, \quad k \in \mathbb{Z}$

9.  $-\log 2 - (\pi + 2k\pi)i, \left( \frac{2}{3}\pi + 2k\pi \right) i, \left( -\frac{2}{3}\pi + 2k\pi \right) i, \quad k \in \mathbb{Z}$

10.  $\pi + 2k\pi, \frac{3}{4}\pi + 2k\pi - \frac{1}{2} \log 2 i, \quad k \in \mathbb{Z}$

11.  $\frac{1}{4} \log 2 + \left( k + \frac{1}{8} \right) \pi + \left( -\frac{1}{4} \log 2 + \left( k + \frac{1}{8} \right) \pi \right) i, \quad k \in \mathbb{Z}.$

12.  $\frac{\pi}{4} + k\pi, \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$