

THE CARDINALITY OF SETS

NOTION OF EQUICARDINALITY

WE SAY THAT THE SETS

X & Y ARE EQUICARDINAL

IF AND ONLY IF

$$\exists f: X \xrightarrow[1-1]{\text{SO}} Y \text{ BIJECTION.}$$

COUNTABLE SETS

SET X IS SAID TO BE COUNTABLE

\Uparrow DEF.

X IS EQUICARDINAL TO THE SET \mathbb{N}
 \uparrow
SET OF NATURAL NUMBERS,

0, 1, 2, ..., n, ...

FOR EXAMPLE,

$$\mathbb{Z}^+ = 1, 2, \dots, n, \dots$$

$\mathbb{Z}^+ \subset \mathbb{N}$ BUT \mathbb{Z}^+ AND \mathbb{N} ARE EQUICARDINAL...

INDEED, THE FUNCTION

$$f: \mathbb{N} \rightarrow \mathbb{Z}^+ \text{ s.t. } f(n) = n+1 \quad \forall n \in \mathbb{N}$$

IS A BIJECTION !!

NOTE THAT THE SET OF "RELATIVE"

INTEGERS \mathbb{Z} IS COUNTABLE.

CANTOR THMS (SPECIAL CASE)

FIRST THEOREM 1) ANY UNION OF A FINITE

OR EVEN A COUNTABLE FAMILY OF COUNTABLE

SET IS STILL A COUNTABLE SET !!

2) ANY CARTESIAN PRODUCT OF

COUNTABLE SETS IS STILL A COUNTABLE SET.

PROVING THE SET OF RATIONALS

LEVEL 1 ... OF ...

NUMBER \mathbb{Q} IS COUNTABLE.



POWER SETS

X SET, $\mathcal{P}(X) = \{A; A \subseteq X\}$.
↑
POWER SET

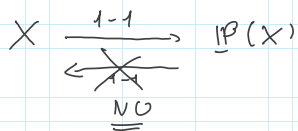
IN THE FINITE CASE, IF $|X| = n$ (FINITE CARD.)
↑
CARDINALITY

$$\Rightarrow |\mathcal{P}(X)| = 2^n > n.$$

II CANTOR THM IN ANY CASE (FINITE OR INFINITE)

THE CARDINALITY OF $\mathcal{P}(X)$ IS STRICTLY

GREATER THAN THE CARDINALITY OF X . !!!



\mathbb{R} REAL NUMBERS,

$\mathbb{R} - \mathbb{Q}$ IRRATIONAL NUMBERS

ARE NOT COUNTABLE, INDEED THEY ARE CONTINUOUS!