Monday, February 12, 2024 10:29 AM

THE CARDINALITY OF SETS

NOTION OF EQUICARDINALITY

WE SAY THAT THE SETS

X & Y ARE EQUICARDINAL

IF AND ONLY IF

$$\exists\, F: X \xrightarrow[1-1]{SU} Y \quad \text{BIJECTION}.$$

COUNTABLE SETS

SET X IS SAID TO BE COUNTABLE

$\Updownarrow$ DEF.

X IS EQUICARDINAL TO THE SET $\mathbb{N}$

$\uparrow$ SET OF NATURAL NUMBERS,

$0, 1, 2, \ldots, n, \ldots$

FOR EXAMPLE,

$\mathbb{Z}^+ = 1, 2, \ldots, n, \ldots$

$\mathbb{Z}^+ \subsetneq \mathbb{N}$ BUT $\mathbb{Z}^+$ AND $\mathbb{N}$ ARE EQUICARDINAL !!!

INDEED, THE FUNCTION

$F: \mathbb{N} \longrightarrow \mathbb{Z}^+$ s.t. $F(n) = n+1 \quad \forall n \in \mathbb{N}$

IS A BIJECTION !!!

NOTICE THAT THE SET OF "RELATIVE" INTEGERS $\mathbb{Z}$ IS COUNTABLE.

CANTOR THMS (SPECIAL CASE)

FIRST THEOREM 1) ANY UNION OF A FINITE OR EVEN A COUNTABLE FAMILY OF COUNTABLE SET IS STILL A COUNTABLE SET !!!

2) ANY CARTESIAN PRODUCT OF COUNTABLE SETS IS STILL A COUNTABLE SET.

COROLLARY THE SET OF RATIONAL

COROLLARY THE SET OF RATIONAL NUMBER $\mathbb{Q}$ IS COUNTABLE.

POWER SETS

X SET, $\mathbb{P}(X) \overset{DEF}{=} \{ A ; A \subseteq X \}$.

POWER SET

IN THE FINITE CASE, IF $|X| = n$ (FINITE CARD.)

CARDINALITY

$$\Rightarrow \quad |\mathbb{P}(X)| = 2^n > n .$$

II CANTOR THM IN ANY CASE (FINITE OR INFINITE)

THE CARDINALITY OF $\mathbb{P}(X)$ IS STRICTLY GREATER THAN THE CARDINALITY OF X. !!!

$$X \xrightarrow{1-1} \mathbb{P}(X)$$

$$X \xleftarrow{\cancel{1-1}} \mathbb{P}(X)$$

NO

$\mathbb{R}$ REAL NUMBERS,

$\mathbb{R} - \mathbb{Q}$ IRRATIONAL NUMBERS

ARE NOT COUNTABLE, INDEED THEY ARE CONTINUOUS !