Wednesday, February 14, 2024 11:21 AM

REPEAT THE QUESTION, PLEASE!!!

WE HAVE TO PROVE:

AT MOST COUNT $\to \{A_i \; ; \; A_i \in \mathcal{E} \overset{DEF}{=} \bigcap_{i \in I} \mathcal{E}_i\}$ $\overset{???}{\Longrightarrow}$

$$\bigcup_{i \in I} A_i \in \mathcal{E} \overset{DEF}{=} \bigcap_{i \in I} \mathcal{E}_i \quad ???$$

NOW, $A_i \in \mathcal{E} = \bigcap_{i \in I} \mathcal{E}_i$

$$\Downarrow$$

$$A_i \in \mathcal{E}_i \quad \forall i \in I$$

$$\Downarrow \quad \mathcal{E}_i \;\; \sigma\text{-ALG.}$$

$$A_i^C \in \mathcal{E}_i \quad \forall i \in I$$

$$\Downarrow \quad \mathcal{E}_i \;\; \sigma\text{-ALG.}$$

$$\bigcup_{i \in I} A_i^C \in \mathcal{E} \overset{DEF}{=} \bigcap_{i \in I} \mathcal{E}_i$$

THE BOREL σ-ALGEBRA

$\mathcal{B}(\mathbb{R})$ $\quad$ $n = 1$.

RECALL

$\mathcal{B}(\mathbb{R})$ IS THE MINIMUM σ-ALGEBRA THAT CONTAINS ALL OPEN SETS!!!

BOREL σ-ALGEBRA

WE RECALL $A \subseteq \mathbb{R}$

$$A \text{ OPEN} \overset{DEF}{\Longleftrightarrow} \forall x \in A \;\; \exists\, r \in \mathbb{R}^+ \text{ S.T. } I(x, r) \subseteq A \;\;!!!! \quad (\infty ?)$$

"CONCRETE VERSION" ???

THM (LINDELÖF LEMMA) (SPECIAL CASE)

ANY $A \subseteq \mathbb{R}$, A OPEN

CAN BE REPRESENTED AS

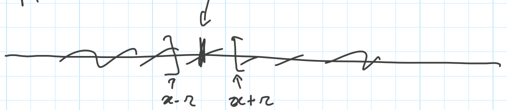
AT MOST COUNTABLE UNION

OF LIMITED OPEN INTERVALS, SAY $]a,b[$.

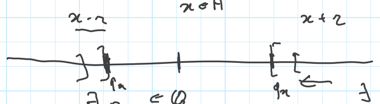
PROOF LET $A \subseteq \mathbb{R}$,

A OPEN $\iff \forall x \in A \ \exists \underbrace{I(x,\delta)}_{\delta \in \mathbb{R}^+} \subseteq A$

$\iff \forall x \in \mathbb{R} \ \exists I(x,r) =]x-r, x+r[\subseteq A$.
$r \in \mathbb{R}^+$



S.T. $]x-r, x+r[\subseteq A$ OPEN



$\exists p_x \in \mathbb{Q}$ S.T. $x-r < p_x < x$

$\exists q_x \in \mathbb{Q}$ S.T. $x < q_x < x+r$

THAT IS $\forall x \in A$

THERE EXIST $p_x \in \mathbb{Q}$, $q_x \in \mathbb{Q}$

SUCH THAT

$\rightarrow x \in]p_x, q_x[$ $\quad p_x, q_x \in \mathbb{Q}$!!!

SO, FOR EVERY $x \in A$

$\rightarrow$ (+) CHOOSE A LIMITED OPEN INTERVAL

$x \in]p_x, q_x[\subseteq A$. $\quad$ $p_x, q_x \in \mathbb{Q}$

CLEARLY, IT FOLLOWS THAT:

$$\bigcup_{x \in A}]p_x, q_x[= A \ !!!$$

HOW MANY ARE THESE

CHOSEN INTERVALS ?!?

BY THE FIRST CANTOR THM $\Rightarrow$

$\Rightarrow$ THEY ARE AT MOST COUNTABLE!

IS IT CLEAR???

STOP QUESTION

BYE BYE

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